केन्द्रीय विद्यालय संगठन

Kendriya Vidyalaya Sangathan

गणित

Mathematics

कक्षा/Class: X

2024-25

विद्यार्थी अध्ययन सामग्री

Student Support Material

संदेश

विद्यालयी शिक्षा में शैक्षिक उत्कृष्टता प्राप्त करना केन्द्रीय विद्यालय संगठन की सर्वोच्च वरीयता है। हमारे विद्यार्थी, शिक्षक एवं शैक्षिक नेतृत्व कर्ता निरंतर उन्नति हेतु प्रयासरत रहते हैं। राष्ट्रीय शिक्षा नीति 2020 के संदर्भ में योग्यता आधारित अधिगम एवं मूल्यांकन संबन्धित उद्देश्यों को प्राप्त करना तथा सीबीएसई के दिशा निर्देशों का पालन, वर्तमान में इस प्रयास को और भी चुनौतीपूर्ण बनाता है।

केन्द्रीय विद्यालय संगठन के पांचों **आंचलिक शिक्षा एवं प्रशिक्षण संस्थान** द्वारा संकलित यह 'विद्यार्थी सहायक सामग्री' इसी दिशा में एक आवश्यक कदम है । यह सहायक सामग्री कक्षा 9 से 12 के विद्यार्थियों के लिए सभी महत्वपूर्ण विषयों पर तैयार की गयी है । केन्द्रीय विद्यालय संगठन की 'विद्यार्थी सहायक सामग्री' अपनी गुणवत्ता एवं परीक्षा संबंधी सामग्री-संकलन की विशेषज्ञता के लिए जानी जाती है और अन्य शिक्षण संस्थान भी इसका उपयोग परीक्षा संबंधी पठन सामग्री की तरह करते रहे हैं । शुभ-आशा एवं विश्वास है कि यह सहायक सामग्री विद्यार्थियों की सहयोगी बनकर सतत मार्गदर्शन करते हुए उन्हें सफलता के लक्ष्य तक पहुंचाएगी ।

शुभाकांक्षा सहित ।

निधि पांडे
आयुक्त, केन्द्रीय विद्यालय संगठन

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CURRICULUM

MATHEMATICS(X)

(CODE NO. 041/241)

Session 2024-25

The Syllabus in the subject of Mathematics has undergone changes from time to time in accordance with growth of the subject and emerging needs of the society. The present revised syllabus has been designed in accordance with National Curriculum Framework 2005 and as per guidelines given in the Focus Group on Teaching of Mathematics which is to meet the emerging needs of all categories of students. For motivating the teacher to relate the topics to real life problems and other subject areas, greater emphasis has been laid on applications of various concepts.

The curriculum at Secondary stage primarily aims at enhancing the capacity of students to employ Mathematics in solving day-to-day life problems and studying the subject as a separate discipline. It is expected that students should acquire the ability to solve problems using algebraic methods and apply the knowledge of simple trigonometry to solve problems of height and distances. Carrying out experiments with numbers and forms of geometry, framing hypothesis and verifying these with further observations form inherent part of Mathematics learning at this stage. The proposed curriculum includes the study of number system, algebra, geometry, trigonometry, mensuration, statistics, graphs and coordinate geometry, etc.

The teaching of Mathematics should be imparted through activities which may involve the use of concrete materials, models, patterns, charts, pictures, posters, games, puzzles and experiments.

Objectives

The broad objectives of teaching of Mathematics at secondary stage are to help the learners to:

- consolidate the Mathematical knowledge and skills acquired at the upper primary stage;
- acquire knowledge and understanding, particularly by way of motivation and visualization, of basic concepts, terms, principles and symbols and underlying processes and skills;
- develop mastery of basic algebraic skills;
- develop drawing skills;
- feel the flow of reason while proving a result or solving a problem;
- apply the knowledge and skills acquired to solve problems and wherever possible, by more than one method;
- to develop ability to think, analyze and articulate logically;
- to develop awareness of the need for national integration, protection of environment, observance of small family norms, removal of social barriers, elimination of gender biases;
- to develop necessary skills to work with modern technological devices and mathematicalsoftware's.
- to develop interest in mathematics as a problem-solving tool in various fields for its beautiful structures and patterns, etc.
- to develop reverence and respect towards great Mathematicians for their contributions to the field of Mathematics;
- to develop interest in the subject by participating in related competitions;
- to acquaint students with different aspects of Mathematics used in daily life;
- to develop an interest in students to study Mathematics as a discipline.

COURSE STRUCTURE CLASS –X

Units	Unit Name	Marks
I	NUMBER SYSTEMS	06
II	ALGEBRA	20
III	COORDINATE GEOMETRY	06
IV	GEOMETRY	15
V	TRIGONOMETRY	12
VI	MENSURATION	10
VII	STATISTICS & PROBABILTY	11
	Total	**80**

UNIT I: NUMBER SYSTEMS

1. REAL NUMBER (15) Periods

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}, \sqrt{3}\ \sqrt{5}$

UNIT II: ALGEBRA

1. POLYNOMIALS (8) Periods

Zeros of a polynomial. Relationship between zeros and coefficients of quadraticpolynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES (15) Periods

Pair of linear equations in two variables and graphical method of their solution, consistency/inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS (15) Periods

Standard form of a quadratic equation $ax^2 + bx + c = 0, (a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS (10) Periods

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY

Coordinate Geometry (15) Periods

Review: Concepts of coordinate geometry, graphs of linear equations. Distance formula.Section formula (internal division).

UNIT IV: GEOMETRY

1. TRIANGLES (15) Periods

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallelto the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their correspondingsides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES (10) Periods

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through thepoint of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1. INTRODUCTION TO TRIGONOMETRY (10) Periods

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0^o and 90^o. Values of the trigonometric ratios of 30^0, 45^0 and 60^0. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES (15) Periods

Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression. (10)Periods

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES (12) Periods

Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES (12) Periods

Surface areas and volumes of combinations of any two of the following: cubes, cuboids,spheres, hemispheres and right circular cylinders/cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS (18) Periods

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY (10) Periods

Classical definition of probability. Simple problems on finding the probability of anevent.

MATHEMATICS-Standard
QUESTION PAPER DESIGN
CLASS – X (2023-24)

Time: 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	43	54
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	18	22
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-Basic
QUESTION PAPER DESIGN
CLASS – X (2023-24)

Time: 3Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	60	75
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	12	15
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	8	10
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

REAL NUMBERS

IMPORTANT POINTS:

The Fundamental Theorem of Arithmetic

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Prime and Composite numbers

A prime number is a number which has only two factors i.e. one and itself whereas the composite number is a number which has more than two factors.

HCF and LCM of numbers

HCF is the highest common factor also known as GCD i.e. greatest common divisor.

LCM of two numbers is their least common multiple.

Property of HCF and LCM of two positive integers 'a' and 'b':

HCF (a, b)×LCM (a, b) = a ×b

HCF and LCM by Prime factorization method

➢ HCF (a, b) = Product of the smallest power of each common prime factor in the numbers.

➢ LCM (a, b) = Product of the greatest power of each prime factor, involved in the numbers.

Multiple Choice Questions

Choose the correct answer from the given four options:

1. The ratio between the LCM and HCF of 5, 15, 20 is:

   (a) 9 : 1 (b) 4 : 3 (c) 11 : 1 (d) 12 : 1

   Solution: Answer (d)

2. If sum of two numbers is 1215 and their HCF is 81, then the possible number of pairs of such numbers are

   (a) 2 (b) 3 (c) 4 (d) 5

   Solution: Answer (c)

3. The LCM of smallest two-digit composite number and smallest composite number is:

   a) 12 b) 4 c) 20 d) 44

   Solution: Answer (c)

4. If two positive integers a and b are written as $a=x^3y^2$ and $b= xy^3$, where x and y are prime numbers , then the result obtained by dividing the product of the positive integers by the LCM(a,b) is

   (a) xy (b) xy^2 (c) $x^3 y^3$ (d) x^2y^2

   Solution: Answer (b)

5. Two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).

Statement A (Assertion): If product of two numbers is 12960 and their HCF is 12, then their LCM is 1080.

Statement R(Reason): HCF is always a factor of LCM

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A)

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A)

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Solution: Answer (b)

SA (TYPE I)

1. Three bells ring at intervals of 4, 7 and 14 minutes. All three rang at 6 AM. When will they ring together again?

Solution: To find the time interval at which bells will ring again, we have to find LCM of three time intervals.

$4 = 2 \times 2 = 2^2$

$7 = 7^1$

$14 = 2^1 \times 7^1$

LCM $= 2^2 \times 7^1 = 28$

Hence, all the three bells will ring together again at 6:28 am

2. Given that $\sqrt{2}$, $\sqrt{5}$ are irrational, prove that $\sqrt{5} + \sqrt{2}$ is irrational.

Solution: Let us suppose that $\sqrt{5} + \sqrt{2}$ is rational.

Let $\sqrt{5} + \sqrt{2} = a$, where a is rational.

Therefore, $\sqrt{2} = a - \sqrt{5}$

Squaring on both sides, we get

$2 = a^2 + 5 - 2\sqrt{5}a$.

Therefore, $\sqrt{5} = \frac{a^2+5-2}{2a}$

which is a contradiction as the right-hand side is a rational number while $\sqrt{5}$ is irrational.

Hence, $\sqrt{5} + \sqrt{2}$ is irrational.

3. Can the number 6^n , n being a natural number, end with the digit 5? Give reasons.

Solution : No, because $6^n = (2 \times 3)^n = 2^n \times 3^n$, so the only primes in the factorisation of 6^n are 2 and 3, and not 5. Hence, it cannot end with the digit 5.

SA (TYPE II)

1. Three sets of English , Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subjectwise and height of each stack is the same. All the books are of same thickness. Find the number of books in each stack. How many stacks will be there?

Solution: To find the no. of books in each stack, we have to find HCF of 336, 240 and 96.

$336 = 2^4 \times 3 \times 7$

$240 = 2^4 \times 3 \times 5$

$96 = 2^5 \times 3$

HCF(336, 420,96) $= 2^4 \times 3 = 48$.

No. of books in each stack $= 48$

Total no. of stacks $= (336 \div 48) + (240 \div 48) + (96 \div 48)$

$= 7 + 5 + 2 = 14$

2. Prove that $\sqrt{7}$ is an irrational number.

Solution:

Let us assume that $\sqrt{7}$ is rational number.
we can find coprime a and b ($b \neq 0$) such that
$\sqrt{7} = \frac{a}{b}$

$a^2 = 7b^2$ ----------(i)

$\Rightarrow$ 7 divides a^2 .

$\Rightarrow$ 7 divides a

Let $a = 7c$ for some positive integer c.

Putting $a = 7c$ in (i), we get

$49c^2 = 7b^2$

$\Rightarrow 7c^2 = b^2$

$\Rightarrow$ 7 divides b^2 .

$\Rightarrow$ 7 divides b.

Thus , 7 is a common factor of a and b.

This contradicts the fact that a and b are coprime. This contradiction has arisen due to our incorrect assumption.

Hence, $\sqrt{7}$ is an irrational number.

3. Prove that $2 + 3\sqrt{5}$ is an irrational number. It is given that $\sqrt{2}$ is an irrational number.

Solution:

Let us assume that $2 + 3\sqrt{5}$ is rational.

Then, we can find coprime a and b ($b \neq 0$) such that $2 + 3\sqrt{5} = \frac{a}{b}$.

$\Rightarrow \sqrt{5} = \frac{a-2b}{3b}$.

Since a and b are integers, $\frac{a-2b}{3b}$ is rational, and so $\sqrt{5}$ is rational. But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $2+3\sqrt{5}$is rational. So, we conclude that $2+3\sqrt{5}$ is irrational.

LA

1. Prove that $\sqrt{6}$ is an irrational number. Using this result , prove that $(\sqrt{2}+\sqrt{3})^2$ is irrational.

Solution: Let us assume that $\sqrt{6}$ is rational number.
we can find coprime a and b ($b \neq 0$) such that

$$\sqrt{6}=\frac{a}{b}$$

$a^2=6b^2$

From above a^2 is even. If a^2 is even, then a should also be even.

$\Longrightarrow a=2c$

$4c^2=6b^2$

$2c^2=3b^2$

From above $3b^2$ is even. If $3b^2$ is even, then b^2 should also be even and again b is even.

But a and b are coprime and both cannot be even. Hence, assumption was wrong and $\sqrt{6}$ is an irrational number.

$(\sqrt{2}+\sqrt{3})^2 = 2+3+2\sqrt{6}= 5+2\sqrt{6}$

Let us assume that $5+2\sqrt{6}$ is rational.

Then, we can find coprime a and b ($b \neq 0$) such that $5+2\sqrt{6}=\frac{a}{b}$.

$$\Rightarrow \sqrt{6}=\frac{a-5b}{2b}.$$

Since a and b are integers, $\frac{a-5b}{2b}$ is rational, and so $\sqrt{6}$ is rational. But this contradicts the fact that $\sqrt{6}$ is irrational. This contradiction has arisen because of our incorrect assumption that $5+2\sqrt{6}$is rational. So, we conclude that $5+2\sqrt{6}=(\sqrt{2}+\sqrt{3})^2$ is irrational.

CASE STUDY:

1. Shanvi is a Mathematics teacher. She is very innovative and always plan new games to make her students learn concepts. Today, she has planned a prime number game. She announces the number 2 in her class and asked first student to multiply it by a prime number and then pass it to second student. Second student also multiplied it by a prime number and passed it to third student. In this way by multiplying to a prime number the last student got 173250. He told this number to Shanvi in class. Based on the above , answer the following questions:

(i)How many students are in the class?

(ii)(a) What is the greatest prime number used by student?

Or,

(b) What is the smallest prime number used by student?

(iii)Which prime number has been used maximum time?

Solution:
$173250 = 2 \times 3^2 \times 5^3 \times 7 \times 11$

(i) No. of students in the class = 2+3+1+1= 7
(Sum of exponents of all primes except 2 because 2 appears only one time in the prime factorization which was announced by the teacher)
(ii) (a) 11 (b) 3
(iii) 5

PRACTICE QUESTIONS:
Choose the correct answer from the given four options:

1. In a formula racing competition , the time taken by two racing cars A and B to compete one round of the track is 30 minutes and p minutes respectively. If the cars meet again at the starting point for the first time after 90 minutes and HCF (30,p)=15, then the value of p is

   (a) 45 minutes (b) 60 minutes (c) 75 minutes (d) 180 minutes

2. Let p be a prime number and k be a positive integer. If p divides k^2, then which of these is definitely divisible by p ?

   (i) k/2 (ii) k (iii) 7k (iv) k^3

   (a) only ii (b) only i and ii (c) only ii, iii and iv (d) all – i, ii, iii and iv

3. The HCF of k and 93 is 31, where k is a natural number. Which of these can be true for some values of k ?

   (i) K is a multiple of 31 (ii) k is a multiple of 93

   (ii) K is an even number (iv) k is an odd number

(a) only ii and iii (b) only i ,ii and iii (c) only i, iii and iv (d) all – i, ii, iii and iv

4. If $a = 2^3 \times 3$, $b = 2 \times 3 \times 5$, $c = 3^n \times 5$ and LCM(a,b,c) $= 2^3 \times 3^2 \times 5$, then n = ------
   (a) 1 (b) 2 (c) 3 (d) 4
5. A number q is prime factorised as $3^2 \times 7^2 \times b$, where b is a prime number other than 3 and 7. Based on the above information, two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R).
   Assertion (A): q is definitely an odd number.
   Reason (R): $3^2 \times 7^2$ is an odd number.
   (a) Both (A) and (R) are true and (R) is the correct explanation for (A).
   (b) Both (A) and (R) are true but (R) is not the correct explanation for (A).
   (c) (A) is true but (R) is false.
   (d) (A) is false but (R) is true.

SA(TYPE I)

1. A forester wants to plant 66 apple trees, 88 banana trees and 110 mango trees in equal rows(in terms of number of trees). Also, he wants to make distinct roots of the trees (only one type of tree in a row). Find the minimum no. of rows required.
2. The LCM of 6^4, 8^2 and k is 12^4 where k is a positive integer. Find the smallest value of k. Show your steps.
3. The LCM of two numbers is 64699, their HCF is 97 and one of the numbers is 2231.Find the other.

SA(TYPEII)

1. Find the smallest pair of 4-digit numbers such that the difference between them is 303 and their HCF is 101. Show your steps.
2. Prove that the difference of $(7 + 2\sqrt{3})$ and $(3 - 5\sqrt{3})$ is irrational.
3. The sum of LCM and HCF of two numbers is 7380.If the LCM of these numbers is 7340 more than their HCF. Find the product of the two numbers.

LA

1. The sum of LCM and HCF of two numbers is 7380.If the LCM of these numbers is 7340 more than their HCF. Find the product of the two numbers.
2. Prime factorisation of three numbers A, B and C is given below:
   $A = (2^r \times 3^p \times 5^q)$
   $B = (2^p \times 3^r \times 5^p)$
   $C = (2^q \times 3^q \times 5^p)$ such that, $p < q < r$ and p, q, & r are natural numbers..
   ♦ The largest number that divides A, B and C without leaving a remainder is 30.

   ♦ The smallest number that leaves a remainder of 2 when divided by each of A, B and C is 5402.
   Find A, B and C. Show your work.

CASE STUDY

1. The Republic Day parade, first held in 1950, has been a yearly ritual since. The parade marches from the Rashtrapati Bhawan along the Rajpath in New Delhi. Several regiments of the army, navy, and air force, along with their bands, march to India Gate. The parade is presided over by the President of India, who is the Commander-in-Chief of the Indian Armed Forces. As he unfurls the tricolour, the national anthem is played. The regiments of the Armed Forces then start their march past. Prestigious awards like Kirti Chakra, Ashok Chakra, Paramvir Chakra are given out by the President. Nine to twelve different regiments of the Indian Army, in addition to the

Navy and Air Force march towards India Gate along with their bands. Contingents of paramilitary forces and other civil forces also participate in the parade. On 76th republic day parade, Commander Bhawnish is planing for parade of following two group:

(a) First group of Army troops of 624 members behind an army band of 32 members.

(b) Second group of CRPF troops with 468 soldiers behind the 228 members of bikers.

These two groups are to march in the same number of columns. This sequence of soldiers is followed by different states Jhanki which are showing the culture of the respective states.

Based on the above information, answer the following questions:

(i)What is the maximum number of columns in whichthe army troop can march?

(ii)What is the maximum number of columns in whichthe CRPF troop can march?

(iii)(a) What is the maximum number of columns in which total army troop and CRPF troop together can march past?

Or,

(b)What should be subtracted with the numbers of CRPF soldiers and the number of bikers so that their maximum number of columns is equal to the maximum number of columns of army troop?

ANSWER KEY OF PRACTICE QUESTIONS:

MCQ

1. ----(a)2.----(c) 3.----(c) 4……(b) 5.----(d)

SA (TYPE I)

1. 12
2. $2^8 = 256$
3. 2813

SA (TYPE II)

1. 1010 and 1313

2. difference of $(7 + 2\sqrt{3})$ and $(3 - 5\sqrt{3}) = 4 + 7\sqrt{3}$ which is irrational(Proof mandatory)

3. Product of nos. $=$ LCM $\times$ HCF $= 7360 \times 20 = 147200$

LA

1. $p = 1, q = 2, r = 3$
   $A = 600, B = 270, C = 180$

CASE STUDY

1. (i) 16

   (ii)12

   (iii) (a) 4 (b)4 soldiers and 4 bikers

POLYNOMIALS

CONCEPTUAL NOTES

Algebraic Expressions

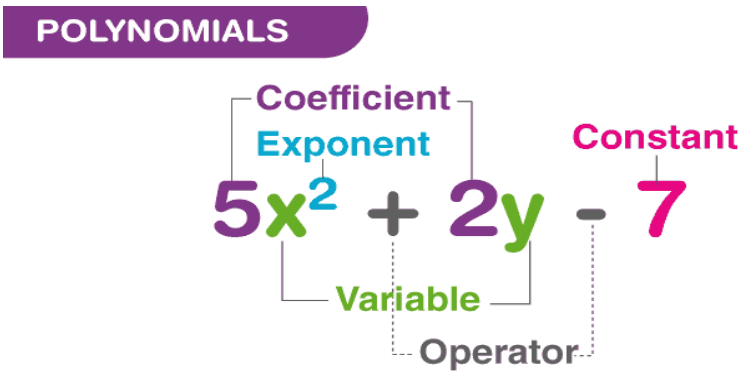
An algebraic expression is an expression made up of variables and constants along with mathematical operators.

An algebraic expression is a sum of terms, which are considered to be building blocks for expressions.

Polynomial

- "Polynomial" comes from the word 'Poly' (Meaning Many) and 'nomial' (in this case meaning Term)-so it means many terms.

- A polynomial is made up of terms that are only added, subtracted or multiplied
- An algebraic expression can have exponents that are rational numbers.
- However, a polynomial is an algebraic expression in which the exponent on any variable is a whole number.



Degree of a Polynomial

For a polynomial in one variable – the highest exponent on the variable in a polynomial is the degree of the polynomial.

Example: The degree of the polynomial x^2+2x+3 is 2, as the highest power of x in the given expression is x^2.

Types Of Polynomials

Polynomials can be classified based on the following.

a) Number of terms

b) Degree of the polynomial.

Types of Polynomials Based on the Number of Terms

a) Monomial – A polynomial with just one term. Example: $2x$, $6x^2$, $9xy$

b) Binomial – A polynomial with two unlike terms. Example: $4x^2+x$, $5x+4$

a) Trinomial – A polynomial with three unlike terms. Example: x^2+3x+4

Types of Polynomials based on Degree

Linear Polynomial

A polynomial whose degree is one is called a linear polynomial.

For example, $2x+1$ is a linear polynomial.

Quadratic Polynomial

A polynomial of degree two is called a quadratic polynomial.

For example, $3x^2+8x+5$ is a quadratic polynomial.

Cubic Polynomial

A polynomial of degree three is called a cubic polynomial**.**

For example, $2x^3+5x^2+9x+15$ is a cubic polynomial.

Zeroes of a Polynomial

A zero of a polynomial **p(x)** is the value of x for which the value of p(x) is 0. If k is a zero of p(x), then **p(k)=0.**

For example, consider a polynomial $p(x)=x^2-3x+2$.

When $x=1$, the value of p(x) will be equal to

$p(1)=1^2-3\times1+2$

$=1-3+2$

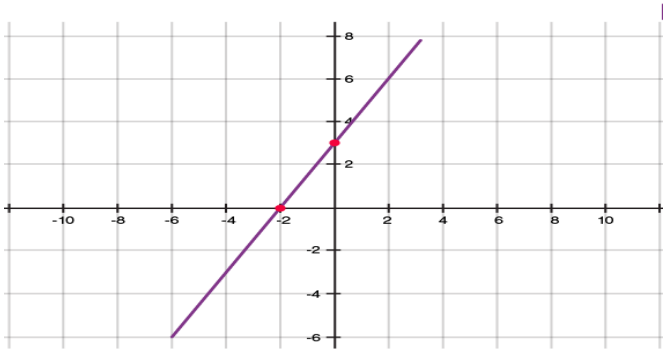
$=0$

Since $p(x)=0$ at $x=1$, we say that 1 is a zero of the polynomial x^2-3x+2

Geometrical Representation and meaning of the zeroes of a Polynomial

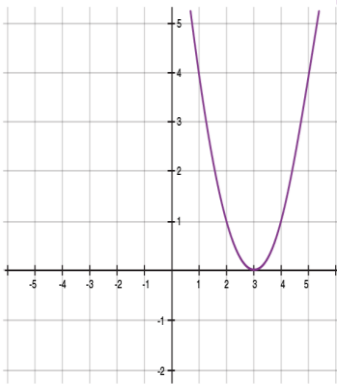
Linear Polynomial

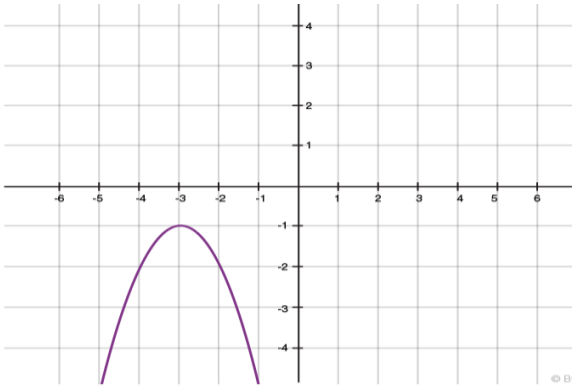
- ✓ The graph of a linear polynomial is a straight line. It cuts the X-axis at exactly one point.
- ✓ If the linear polynomial is represented by $y = ax + b$, then it cuts the x- axis at $\frac{-b}{a}$.



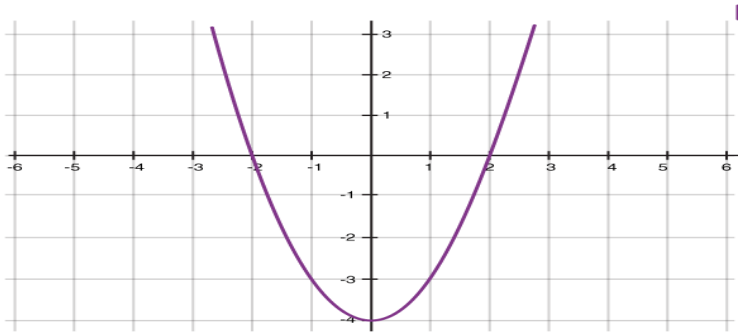
Quadratic Polynomial

- The graph of a quadratic polynomial is a parabola
- It looks like a U, which either opens upwards or opens downwards depending on the value of 'a' in ax^2+bx+c
- If 'a' is positive, then parabola opens upwards and if 'a' is negative then it opens downwards



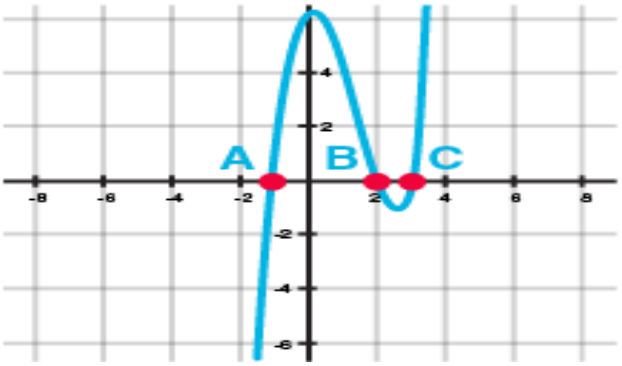


- It can cut the x-axis at 0, 1 or two points.

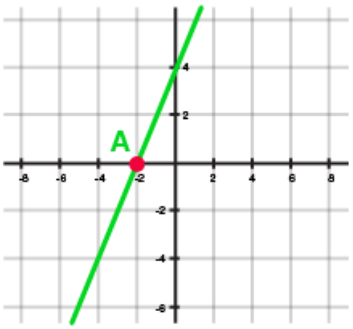


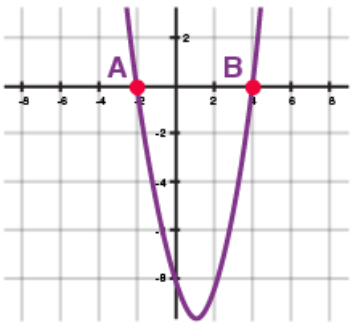
Cubic Polynomial

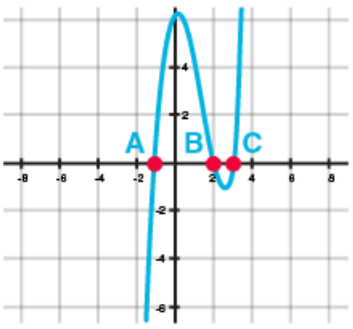
The general form of a cubic polynomial is $ax^3+bx^2+cx+d=0$, where $a\neq0$.



Geometrically, zeros of a polynomial are the points where its graph cuts the x-axis.







(i) One zero (Linear Polynomial) (ii) Two zeros (Quadratic Polynomial) (iii) Three zeros (Cubic Polynomial)

Here A, B and C correspond to the zeros of the polynomial represented by the graphs.

Number of Zeros

In general, a polynomial of degree n has at most n zeros.

- A linear polynomial has one zero.
- A quadratic polynomial has at most two zeros.
- A cubic polynomial has at most 3 zeros.

Factorisation of Polynomials

Quadratic polynomials can be factorized by splitting the middle term.

For example, consider the polynomial $2x^2-5x+3$

Splitting the middle term:

The middle term in the polynomial $2x^2-5x+3$ is -5x. This must be expressed as a sum of two terms such that the product of their coefficients is equal to the product of 2 and 3 (coefficient of x^2 and the constant term)

-5 can be expressed as $(-2)+(-3)$, as $-2\times-3=6=2\times3$

Thus, $2x^2-5x+3 = 2x^2-2x-3x+3$

Now, identify the common factors in individual groups

$2x^2-2x-3x+3 = 2x(x-1)-3(x-1)$

Taking $(x-1)$ as the common factor, this can be expressed as:

$2x(x-1)-3(x-1) = (x-1)(2x-3)$

Relationship between Zeroes and Coefficients of a Polynomial

For Quadratic Polynomial:

If α and β are the roots of a quadratic polynomial ax^2+bx+c, then,

$\alpha + \beta = -b/a$

Sum of zeroes = -coefficient of x /coefficient of x^2

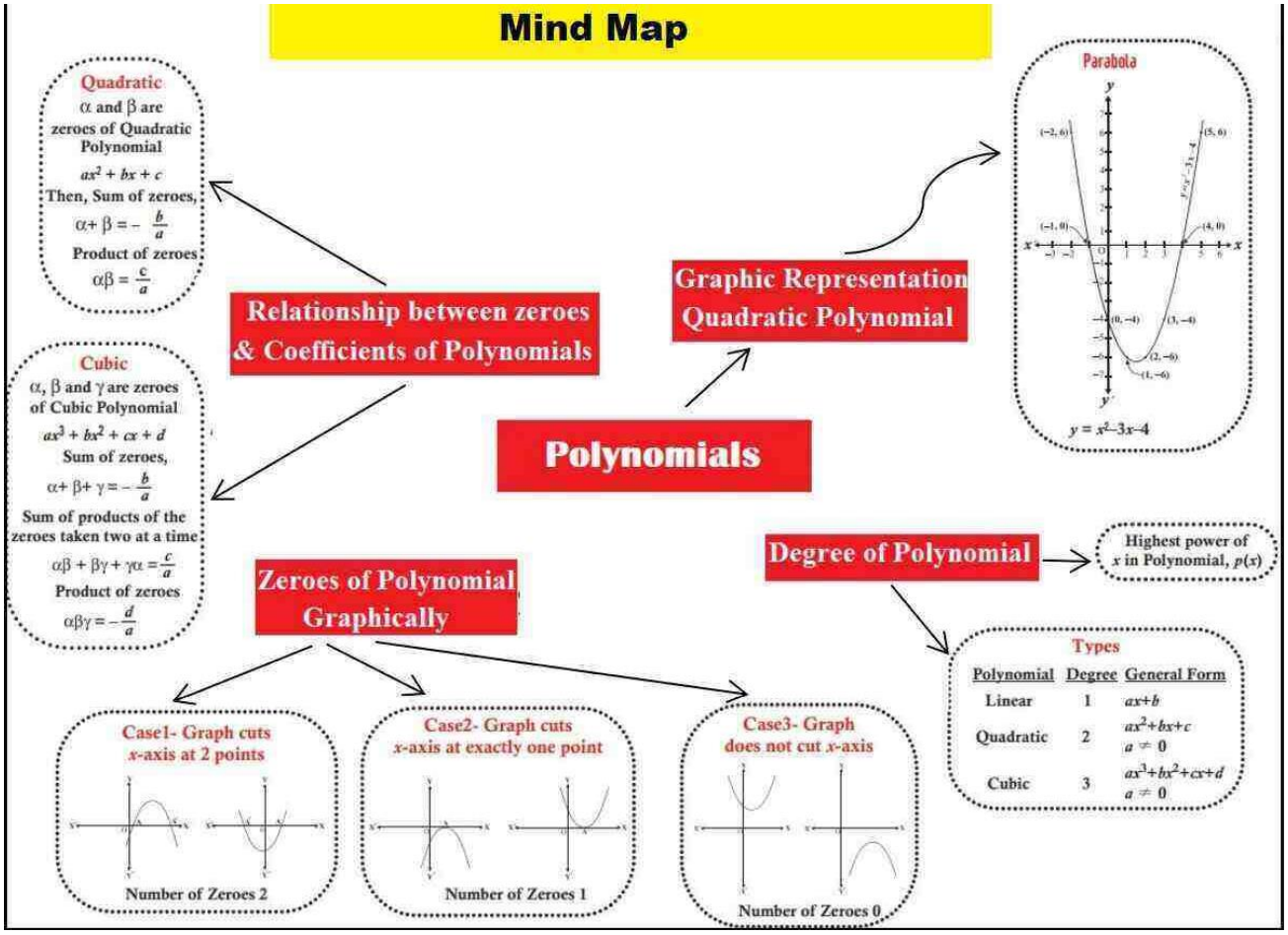
$\alpha\beta = c/a$

Product of zeroes = constant term / coefficient of x^2

Formation of a quadratic polynomial if the roots are given

If α and β are the zeroes of a quadratic polynomial, then the polynomial can be formed as – X^2 - (sum of zeroes) x + (product of zeroes).

$x^2-(\alpha+\beta)x+\alpha\beta$



IMPORTANT POINTS

- If α and β are the zeroes of a quadratic polynomial, then the polynomial can be formed as –
  X^2 - (sum of zeroes) x + (product of zeroes).
  $x^2 - (\alpha+\beta)x + \alpha\beta$
- If α and β are the roots of a quadratic polynomial ax^2+bx+c, then,
  $\alpha + \beta = -b/a$ Sum of zeroes = -coefficient of x /coefficient of x^2
  $\alpha\beta = c/a$ Product of zeroes = constant term / coefficient of x^2
- A linear polynomial has one zero, a quadratic polynomial has at most two zeros. and a cubic polynomial has at most 3 zeros.
  Zeros of a polynomial p(x) are the points where its graph cuts the x-axis.
- A zero of a polynomial **p(x)** is the value of x for which the value of p(x) is 0. If k is a zero of p(x), then **p(k)=0.**

MULTIPLE CHOICE QUESTIONS (SOLVED)

1. If $p(x) = ax^2 + bx + c$, then $-\frac{b}{a}$ is equal to
   (a) 0 (b) 1 (c) product of zeroes (d) sum of zeroes

Ans – (d) sum of zeroes

2. If the square of the difference of the zero of the quadratic polynomial $x^2 + px + 45$ is equal to 144, find the value of p.
(a) 18 (b) -18 (c) both a & b (d) none of these

Ans – (c) both a & b

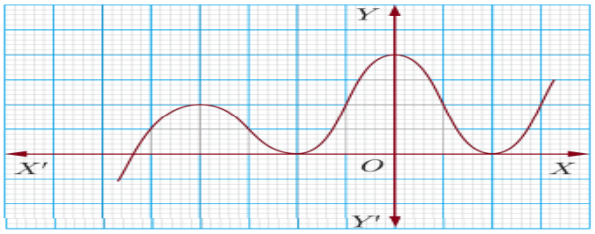
3. If the sum of the zeroes of the quadratic polynomial $ky^2 + 2y - 3k$ is equal to twice their product, find the value of k.
(a) 3 (b) 1/3 (c) 2 (d) 1

Ans – (b) 1/3
4. If α,β are zeroes of the polynomial $f(x) = x^2 - p(x+1) - c$, then find the value of $(\alpha + 1)(\beta + 1)$.
(a) 1 + c (b) 1/c (c) c (d) 1 – c
Ans – (d) 1 - c
Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

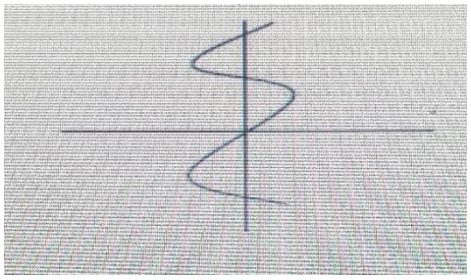
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

5. **Assertion:** The graph y=f(x) is shown in figure, for the polynomial f(x). The number of zeroes of f(x) is 3.

Reason: The number of zero of the polynomial f(x) is the number of point of which f(x) cuts or touches the axes.



MULTIPLE CHOICE QUESTIONS (UNSOLVED)
1. Twice the product of the zeroes of the polynomial $23x^2 - 26x + 161 = 14p$. Find p.
(a) 2 (b) 0 (c) 1 (d) 201
2. Find the number of zeroes of the polynomial from the graph given below



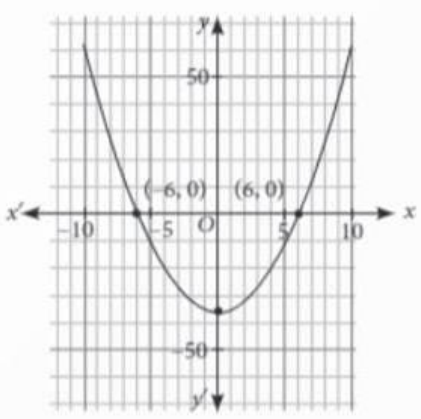
(a) 0 (b) 1 (c) 3 (d) 4
3. If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then
(a) a = -7, b = -1 (b) a = 5, b = -1 (c) a = 2, b = -6 (d) a = 0, b = -6
4. If one zero of the quadratic polynomial $4x^2 - 8kx + 8x - 9$ is negative of the other, then find the zeroes of $kx^2 + 3kx + 2$
(a) 0 (b) 1 (c) 2 (d) 4
Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true
5. Assertion (A) : Both zeroes of the quadratic polynomial $x^2 - 2kx + 2$ are equal in magnitude but opposite in sign then value of k is ½.
Reason (R) : Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$

SHORT – ANSWER TYPE QUESTIONS (2 MARKS) (SOLVED)

1. While visiting his village, Sohan saw a honeycomb and asked his father what is that. He replied that it's a honeycomb made by honey bees to store honey. Also, he told him that the shape of the honeycomb formed is parabolic. The mathematical representation of the honeycomb structure is shown in the graph.



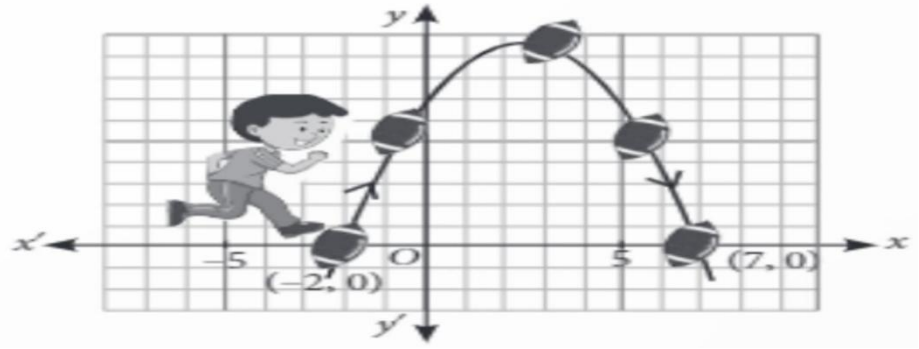
Find the expression of the polynomial represented by the graph.

Solution

Since the graph of the polynomial cuts the x – axis at (-6,0) and (6,0). So, the zeroes of the polynomial are -6 and 6.
Therefore, required polynomial is given by –

X^2 - (sum of zeroes) x + (product of zeroes).
$= x^2 - (\alpha+\beta)x + \alpha\beta$
$= x^2 - (-6 + 6)\, x + (-6)(6)$
$= x^2 - 36$

2. In a soccer match, the path of the soccer ball in a kick is recorded as shown in the following graph.

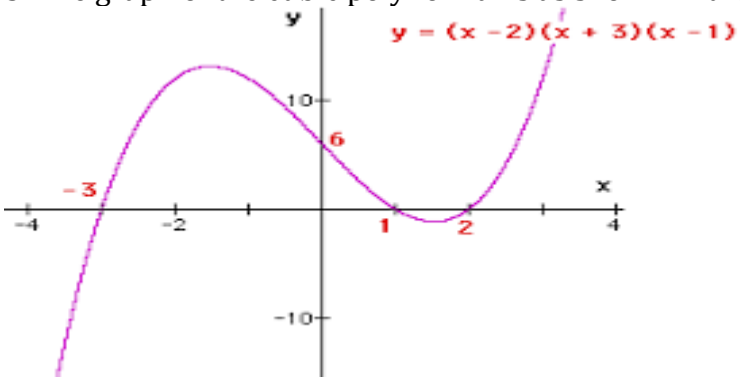


For what value of 'x', the value of the polynomial $f(x) = (x-3)^2 + 9$ is 9?

Solution

We have f(x) = $(x-3)^2 + 9$
Now, $9 = (x-3)^2 + 9$
$\Rightarrow 9 - 9 = (x-3)^2$
$\Rightarrow 0 = (x-3)^2$
$\Rightarrow x - 3 = 0$
$\Rightarrow x = 3.$

3. The graph of the cubic polynomial is as shown in the graph.



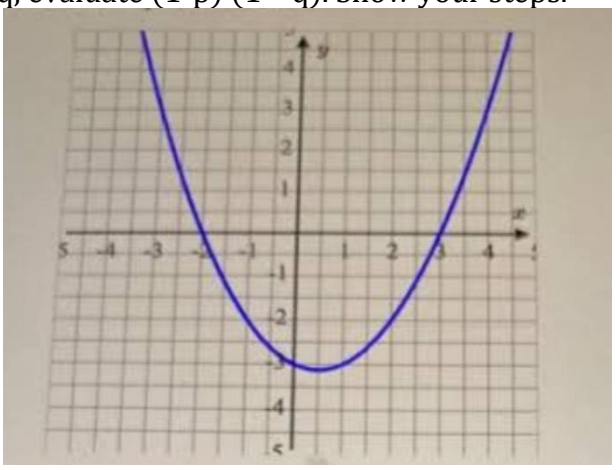
Find the sum of the zeroes of the given polynomial.

Solution

The zeroes of the polynomial as shown in the graph are –
-3, 1 and 2
Therefore, sum of the zeroes = (-3) + 1 + 2
= -3 + 3
= 0

SHORT – ANSWER TYPE QUESTIONS (2 MARKS) (UNSOLVED)

1. If p and q are zeroes of the polynomial $2x^2 + 5x - 4$. Without finding the actual values of p and q, evaluate (1-p) (1 – q). Show your steps.



2. Write a quadratic polynomial whose sum of zeroes is less than that of the polynomial shown in the graph above.

3. The two zeroes of the polynomial $p(x) = 2x^2 - 6x - 3$ are of the form $\frac{3\pm\sqrt{k}}{2}$, where k is a real number.
Use the relationship between the zeroes and the coefficients of a polynomial to find the value of k. Show your steps.

SHORT – ANSWER TYPE QUESTIONS (3 MARKS) (SOLVED)

1. If $p(x) = ax^2 - 8x + 3$,where 'a' is a non-zero real number. One zero of p(x) is three times the other zero.
(a) Find the value of a. Show your work.
(b) What is the shape of the graph of p(x)? Give reason for your answer.

Solution

(a) Assumes the roots of p(x) to be α and β and write the relation $\alpha = 3\beta$
The sum of the zeroes, $\alpha + \beta = -b/a$
$\Rightarrow 3\beta + \beta = -(-8)/a$
$\Rightarrow 4\beta = 8/a$
$\Rightarrow \beta = 2/a$
$\Rightarrow \alpha = 3\beta = 6/a$

Product of the roots, $\alpha\ \beta$ = c/a

$$=>\frac{6}{a}\times\frac{2}{a}=\frac{3}{a}$$

$$=>\frac{12}{a^2}=\frac{3}{a}$$

$$=>a=4.$$

(b) Since 'a' is positive, the graph of p(x) is open upward parabola.

2. If one zero of the polynomial $5z^2 + 13z - p$ is reciprocal of the other, then find p.

Solution

Let the first root be α.

Therefore, the second root will be $1/\alpha$.

Product of the roots $\alpha * 1/\alpha$ = c/a

$=> 1 = -p/5 \quad => p = -5$

3. Find the quadratic polynomial, the sum of whose zeroes is 8 and their product is 12. Hence, find the zeroes of the polynomial.

Solution

Required polynomial is given by –

X^2 - (sum of zeroes) x + (product of zeroes).

$= x^2 - (\alpha+\beta)x + \alpha\beta$

$= x^2 - 8x + 12$

Zeroes of the polynomial $x^2 - 8x + 12$

$= x^2 - 6x - 2x + 12$

$= x(x - 6) - 2(x - 6)$

$= (x - 6)(x - 2)$

Hence the zeroes of the polynomial are 2 and 6.

SHORT – ANSWER TYPE QUESTIONS (3 MARKS) (UNSOLVED)

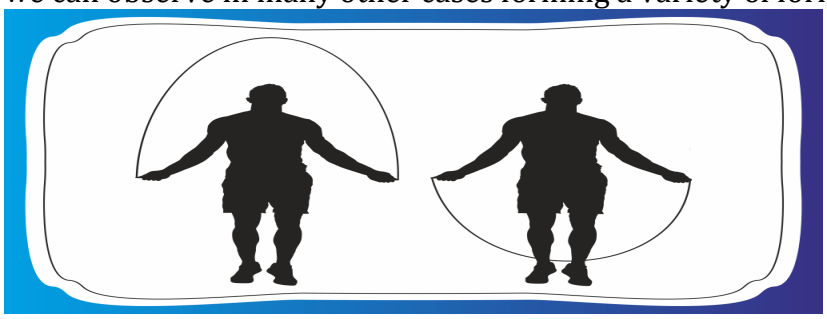
1. Find the zeroes of the quadratic polynomial $4x^2 - 4x - 3$ and verify the relation between the zeroes and its coefficients.

2. If α and β are zeroes of the quadratic polynomial $x^2 - 6x + a$; find the value of 'a' if $3\alpha + 2\beta = 20$.

3. If α and β are zeros of the polynomial $2x^2 - 5x + 7$, then find the value of $\alpha^{-1} + \beta^{-1}$.

CASE STUDY BASED QUESTION (4 MARKS) (SOLVED)

1. During the skipping through skipping rope, it looks like in the form of a parabola. It is a natural example of parabolic shape which is represented by a quadratic polynomial. Similarly, we can observe in many other cases forming a variety of forms of different parabolas.



(a) In the standard form of quadratic polynomial, $ax^2 + bx + c$. What are the conditions for a, b and c.

Solution

'a' is a non – zero real number whereas b and c can be any real number.

(b) The graph of the quadratic polynomial $x^2 - 1$ intersects the x-axis at how many points? What are the points?

Solution

The graph of the quadratic polynomial $x^2 - 1$ intersects the x-axis at two points. Points are (-1,0) and (1,0)

(c) If α and $-\alpha$ are the zeroes of the quadratic polynomial $2x^2 - 3(k - 4)x - 8$, find k.

Solution

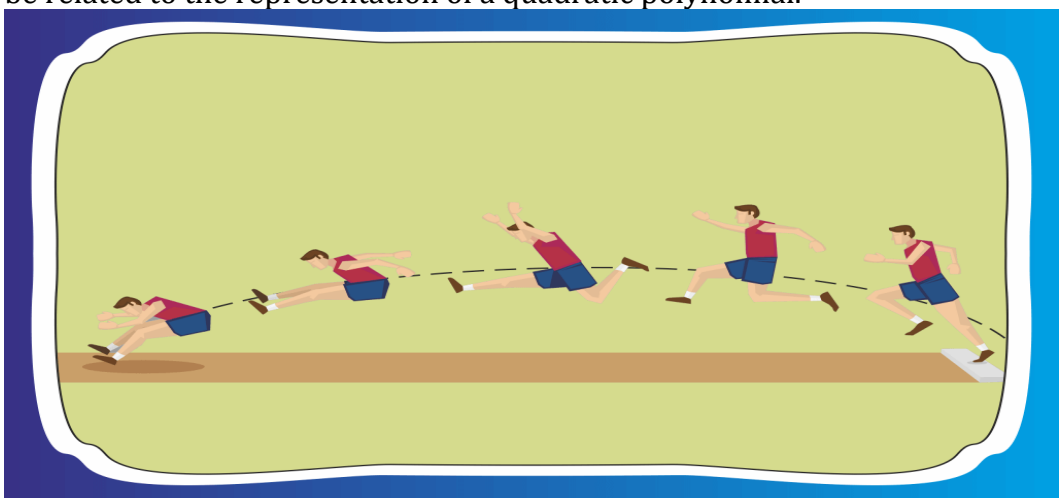
Sum of the zeroes $\alpha + (-\alpha) = -b/a$

$=> 0 = 3(k-4)/2$

$=> 0 = k - 4$

$=> k = 4$

2. Observe the position of the athlete taking long jump. He used to follow every time a particular shape of the path. In the figure, a student can observe that the different positions can be related to the representation of a quadratic polynomial.



(a) What is the name of the path formed by different positions of the athlete.

Ans – Parabola

(b) In the above case, if the quadratic polynomial is represented by $ax^2 + bx + c$, then what can you say about the value of 'a'?

Ans – As the parabola open downwards, the value of 'a' will be less than 0. $(a < 0)$

(c) If the sum and product of the zeroes of the quadratic polynomial $ax^2 + bx + c$ are equal. What is the relation between 'b' and 'c'?

Solution – Given polynomial is $ax^2 + bx + c$.

Sum of the zeroes $= -b/a$

Product of the zeroes $= c/a$

According to question, $-\frac{b}{a} = \frac{c}{a}$

$=> -b = c \quad => b + c = 0$

CASE STUDY BASED QUESTION (4 MARKS) (UNSOLVED)

1. Two Friends Geeta and Sita were playing near the river. So, they decide to play a game in which they have to throw the stone in the river, and whoever will throw the stone at maximum distance, win the game. Geeta Starts first and throws the stone in the river. During her throw, her hand was making an angle of 60° with the Horizontal plane. Sita throws at 45°.

(a) What is the shape of trajectory formed by stone when Geeta & Sita throw it in the river.

(b) If we make a mathematical equation of the path followed by stone when Geeta & Sita threw it in the river, then the resulting mathematical equation would be?

(c) If the zeroes of the quadratic polynomial $x^2 + (a+1)x + b$ are 2 and -3, then find 'a' and 'b'.

2. Prachi was playing with a slinky spring dog toy and asked her brother Rick, what is the shape thus formed called. Rick explained her that the shape formed is a parabola. He also explained her that parabola is the graphical representation of a quadratic polynomial.

(a) What is the general form of the polynomial representing the parabolic graph?
(b) Prachi drawn a parabola passing through (-4,3), (-1,0), (1,8), (0,3), (-3,0) and (-2,-1) on a graph paper. Find the zeroes of the polynomial representing the graph.
(c) Find the sum and product of the roots of the quadratic polynomial $5x\ (x - 6)$.

2. PAIRS OF LINEAR EQUATIONS IN TWO VARIABLES

IMPORTANT POINTS:

- **Linear Equation in Two Variables**- An equation which can be put in the form

  $$ax + by + c = 0$$

  where a,b,c are Real Numbers & a,b are not both zero, is called a Linear Equation in Two Variables x & y.

  Example:- $2x + 5y - 6 = 0$
  $[a = 2, b = 5, c = -6]$

- **General Form for a Pair of Linear Equations in Two Variables x & y-**

  $a_1x + b_1y + c_1 = 0$ [a_1, b_1, c_1 are Real Numbers & a_1, b_1 are not both zero]
  $a_2x + b_2y + c_2 = 0$ [a_2, b_2, c_2 are Real Numbers & a_2, b_2 are not both zero]

- **Method of Finding the Solution for a Pair of Linear Equations in Two Variables x & y-**

  i) Graphical Method
  ii) Algebraic Method
  a) Substitution Method
  b) Elimination Method

- **Graphical Method**

Plot the graph of the first equation and then graph of the second equation on the same rectangular coordinate system. The following three cases may arise.

- Case 1 - If the lines intersect at a point, then the given system has a unique solution given by the coordinates of the point of intersection.
- Case 2 - If the lines are coincident, then the system is consistent and has infinitely many solutions. In this case, every solution of one of the equations is a solution of the system.
- Case 3 - If the lines are parallel, then the given system of equations is inconsistent i.e., it has no solution.

- **Substitution Method**

- Step 1 : Find the value of one variable, say y in terms of the other variable, i.e., x from either equation, whichever is convenient.
- Step 2 : Substitute this value of y in the other equation, and reduce it to an equation in one variable, i.e., in terms of x, which can be solved.
- Step 3 : Substitute the value of x (or y) obtained in Step 2 in the equation used in Step 1 to obtain the value of the other variable.

- **ELIMINATION METHOD**

- Step 1 : First multiply both the equations by some suitable non-zero constants to make the coefficients of one variable (either x or y) numerically equal.
- Step 2 : Then add or subtract one equation from the other so that one variable gets eliminated. If you get an equation in one variable, go to Step 3.
  If in Step 2, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions.
  If in Step 2, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.
- Step 3 : Solve the equation in one variable (x or y) so obtained to get its value.
- Step 4 : Substitute this value of x (or y) in either of the original equations to get the value of the other variable.
- **Graphical & Algebraic Interpretation:**

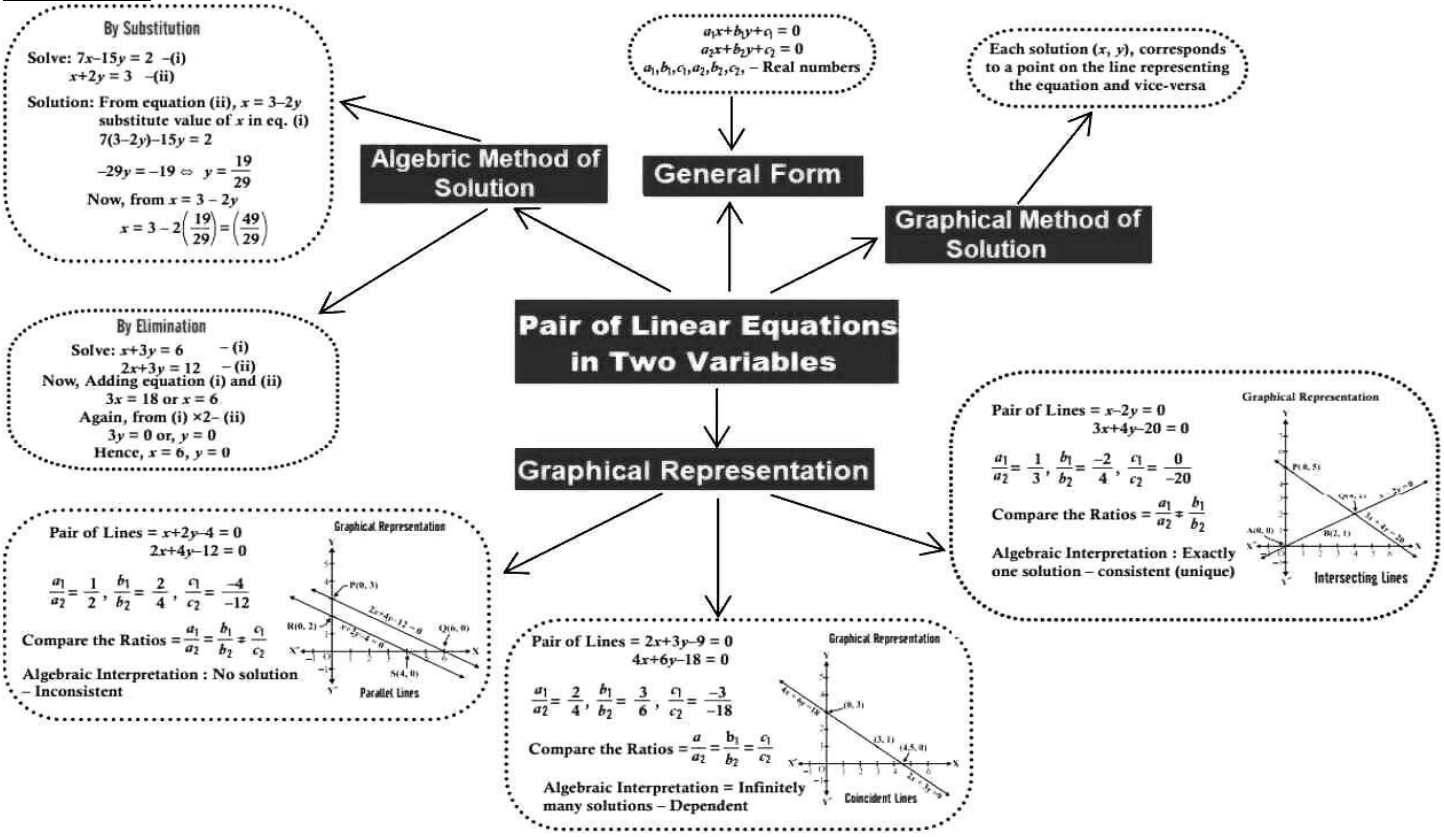
Pair of Linear Equations	Algebraic Condition	Graphical Interpretation	Algebraic Interpretation	Consistency	Example
$a_1x + b_1y + c_1 = 0$ $a_2x + b_2y + c_2 = 0$	$a_1/a_2 \neq b_1/b_2$	Intersecting Lines	Unique Solution	Consistent	$x - 2y = 0$ $3x + 4y - 20 = 0$
	$a_1/a_2 = b_1/b_2 = c_1/c_2$	Coincident Lines	Infinitely Many Solutions	Dependent Consistent	$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$
	$a_1/a_2 = b_1/b_2 \neq c_1/c_2$	Parallel Lines	No Solution	Inconsistent	$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$

- **Consistent Pair of Linear Equations**- A pair of linear equations in two variables, which has a solution, is called a Consistent pair of Linear Equations.

- **Inconsistent Pair of Linear Equations-** A pair of linear equations which has no solution, is called an Inconsistent Pair of Linear Equations.

- **Dependent pair of Linear Equations-** A pair of linear equations which are equivalent & has infinitely many distinct common solutions, is called a Dependent pair of Linear Equations (always consistent).

MIND MAP



- **SOLVED QUESTIONS:**

MCQ (1 MARK)

1. The solution of the equations x-y=2 and x+y=4 is:
   (a) 3 and 1 (b) 4 and 3 (c) 5 and 1 (d) -1 and -3

   Ans: (a) 3 and 1

2. If the lines 3x+2ky – 2 = 0 and 2x+5y+1 = 0 are parallel, then what is the value of k?
   (a) 4/15 (b) 15/4 (c) 4/5 (d) 5/4

   Ans: (b) 15/4
3. The pair of equations 3x – 5y = 7 and – 6x + 10y = 7 have
   (a) a unique solution (b) infinitely many solutions
   (c) no solution (d) two solutions
   Ans: (c) no solution
4. If a pair of linear equations is consistent, then the lines will be
   (a) always coincident (b) parallel
   (c) always intersecting (d) intersecting or coincident
   Ans: (d) intersecting or coincident
5. Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct option:
   Assertion: A pair of linear equations has no solution (s) if it is represented by intersecting lines graphically.
   Reason: If the pair of lines are intersecting, then the pair has unique solution and is called consistent pair of equations.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Ans: (d) Assertion (A) is false but reason (R) is true.

SA (2 MARKS)

6. On comparing the ratios a1/a2, b1/b2, and c1/c2, find out whether the following pair of linear equations are consistent or inconsistent.

   $5x + 7y = 5$; $2x – 3y = 7$

   Ans: Given : $5x + 7y = 5$ or $5x + 7y – 5 = 0$

   and $2x – 3y = 7$ or $2x – 3y – 7 = 0$

   Comparing the above equations with

   $a_1x + b_1y + c_1=0$

   $a_2x + b_2y + c_2 = 0$

   We get,

   $a_1 = 5, b_1 = 7, c_1 = -5$

   $a_2 = 2, b_2 = -3, c_2 = -7$

   $a_1/a_2 = 5/2, b_1/b_2 = 7/-3, c_1/c_2 = -5/-7 = 5/7$

   Since, $a_1/a_2 \neq b_1/b_2$ the lines intersect each other at a point and have only one possible solution. Hence, the equations are consistent.

7. Solve the following pair of linear equations: $s + t = 15$

   $2s - 3t = 5$

   Ans: Given, $s + t = 15$ and $2s – 3t = 5$ are the two equations.

   From 1st equation, we get,

   $s = 15 – t$

   Now, put the value of x in second equation to get,

   $2(15 – t) – 3t = 5$

   $30 – 2t – 3t = 5$

   $5t = 30 – 5 = 25$

   Or, $t = 5$

   By the value of t, we can now find the value of s;

   $\because s = 15 – t$

   $\therefore s = 15 – 5$

   Or, $s = 10$

   Hence, $s = 10$ and $t = 5$.

8. The cost of 2 kg of apples and 1kg of grapes on a day was found to be Rs.160. After a month, the cost of 4 kg of apples and 2 kg of grapes is Rs.300. Represent the situation algebraically. Form a pair of linear equations in two variables based on the above situation.

   Ans: Let the cost of 1 kg of apples be ‘Rs. x’.

   And, let the cost of 1 kg of grapes be ‘Rs. y’.

   According to the question, the pair of linear equations is

   $2x + y = 160$

   $4x + 2y = 300$

SA (3 MARKS)

9. Find the value(s) of k so that the pair of equations $x + 2y = 5$ and $3x + ky + 15 = 0$ has a unique solution.

   Ans: $x + 2y = 5$

$3x + ky + 15 = 0$

Also, given that the pair of equations has a unique solution.

Comparing the given equations with standard form,

$a_1 = 1, b_1 = 2, c_1 = -5$

$a_2 = 3, b_2 = k, c_2 = 15$

Condition for unique solution is:

$a_1/a_2 \neq b_1/b_2$

$1/3 \neq 2/k$

$k \neq (2)(3)$

$k \neq 6$

Thus, for all real values of k except 6, the given pair of equations has a unique solution.

10. A fraction becomes 5/7 if 2 is added to both the numerator and the denominator. If, 1 is subtracted from both the numerator and the denominator it becomes 1/2. Find the fraction.

Ans: Let the fraction be x/y.

According to the question,

$(x + 2)/(y + 2) = 5/7$

$7x + 14 = 5y + 10$

$7x – 5y = -4$ …….. (1)

$(x – 1)/(y – 1) = 1/2$

$2x – 2 = y – 1$

$2x – y = 1$ ……. (2)

From (1), we get

$x = (-4 + 5y)/7$ …….. (3)

Substituting the value of x in (2), we get

$2[(-4 + 5y)/7] – y = 1$

$-8 + 10y – 7y = 7$

$3y = 15$

$y = 5$ …… (4)

Substituting the value of y in (3), we get

$x = (-4 + 25)/7 = 21/7 = 3$

Hence, the fraction is 3/5.

11. Solve the equations $x + 2y – 4 = 0$ and $2x + 4y – 12 = 0$ graphically.

Ans: Given,

$x + 2y – 4 = 0$….(i)

2x + 4y – 12 = 0….(ii)

From (i),

x + 2y = 4

2y = 4 – x

y = (4 – x)/2

x	0	2	4
y	2	1	0

From (ii),

2x + 4y = 12

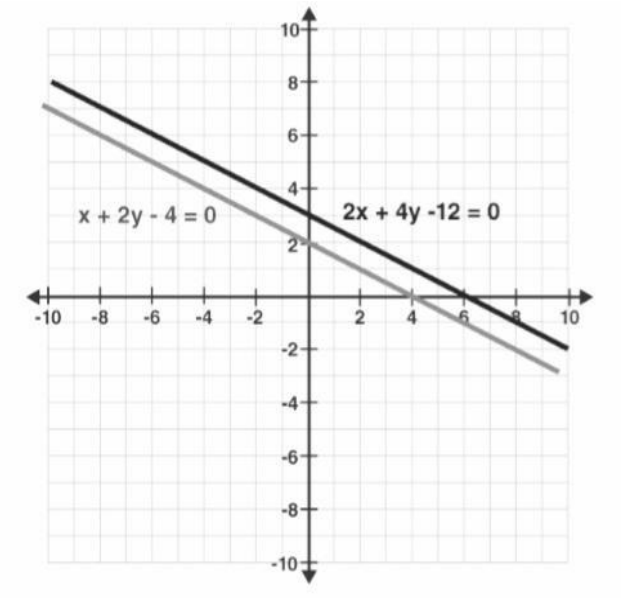
x + 2y = 6

2y = 6 – x

y = (6 – x)/2

x	0	2	4
y	3	2	1

Plotting the points on the graph, we get;



Here, the lines represent the given pair of linear equations are parallel.
Thus, there is no solution to the given pair of linear equations.

LA (5 MARKS)

12. On reversing the digits of a two digit number, number obtained is 9 less than three times the original number. If difference of these two numbers is 45, find the original number.

Ans: Let unit's place digit be x and ten's place digit bey.

∴ Original number = x + 10y Reversed number = 10x + y

According to the Question,

$10x + y = 3(x + 10y) – 9$

$\Rightarrow 10x + y = 3x + 30y – 9$

$\Rightarrow 10x + y – 3x – 30y = -9$

$\Rightarrow 7x – 29y = -9$ …(i)

$10x + y – (x + 10y) = 45$

$\Rightarrow 9x – 9y = 45$

$\Rightarrow x – y = 5$ …[Dividing both sides by 9].....(ii)

Now, multiplying equation (ii) by 7, we get

$7x – 7y = 35$…….(iii)

Subtracting Equation (i) from equation (iii),

$–7y + 29y = 35 + 9$

$\Rightarrow 22y = 44$

$\Rightarrow y = 2$

Putting the value of y in (ii),

$x = 5 + 2 = 7$

∴ Original number = $x + 10y = 7 + 10(2) = 27$

CASE STUDY (4 MARKS)

13. Mr Manoj Jindal arranged a lunch party for some his friends. The expense of the lunch are partly constant and partly proportional to number of guests. The expenses amount to Rs. 650 for 7 guests and Rs. 970 for 11 guests. Denote the constant by Rs x and proportional expense per person by Rs y & answer the following questions

i) Form the pair of linear equations in two variables on the basis of above situation.

ii) What is the fixed (or constant) expense for the party?

iii) If there would be 15 guests at the party, what amount Mr jindal has to pay ?

(1 + 2 + 1)

Ans: i) Here, Constant Expense = Rs. x

Proportional Expense = Rs. y

So, the pair of linear equations in two variables is

$x + 7y = 650$ ……..(1)

$x + 11y = 970$ …….(2)

ii) Subtracting Equation (1) from Equation (2), we get

$11y – 7y = 970 – 650$

$4y = 320$

$y = 80$

Now putting the value of y in Equation (1),

$x + 7 \times 80 = 650$

$x + 560 = 650$

$x = 650 - 560$

$= 90$

So, Fixed Expense = Rs. 90

iii) For 15 guests, Mr Jindal has to pay

$= 90 + 15 \times 80$

$= 90 + 1200$

$=$ Rs. 1290

- **UNSOLVED QUESTIONS**

MCQ (1 MARK)

1. The pairs of equations $9x + 3y + 12 = 0$ and $18x + 6y + 26 = 0$ have
   (a) Unique solution (b) Exactly two solutions
   (c) Infinitely many solutions (d) No solution

2. The solution of $4/x+3y=14$ and $3/x-4y=23$ is:

   (a) ⅕ and -2 (b) ⅓ and ½ (c) 3 and ½ (d) 2 and ⅓

3. The value of c for which the pair of equations $cx - y = 2$ and $6x - 2y = 3$ will have infinitely many solutions is
   (a) 3 (b) -3 (c) -12 (d) no value

4. A fraction becomes 1/3 when 1 is subtracted from the numerator and it becomes 1/4 when 8 is added to its denominator. The fraction obtained is:
   (a) 3/12 (b) 4/12 (c) 5/12 (d) 7/12

5. **Directions:** In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct option:
Assertion: The value of k for which the system of linear equations $kx-y=2$ and $6x-2y=3$ has a unique solution is 3.
Reason: The graph of linear equations $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ gives a pair of intersecting lines if $a_1/a_2 \neq b_1/b_2$
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(C) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

SA (2 MARKS)

6. Solve the following pair of linear equations: $3x - y = 3$
$9x - 3y = 9$

7. On comparing the ratios a_1/a_2, b_1/b_2, and c_1/c_2, find out whether the following lines are intersecting, coincident or parallel .
$2x - 3y = 8$; $4x - 6y = 9$

8. Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Form a pair of linear equations in two variables based on the above situation.

SA (3 MARKS)

9. Solve the following pair of equations:
$49x + 51y = 499$

$51x + 49y = 501$

10. Three years hence, the age of Rani will be three times that of his son. Five years ago, Rani's age was seven times that of his son. What are their present ages?

11. Solve the following pair of equations graphically:

$7x - 15y = 2$

$x + 2y = 3$

LA (5 MARKS)

12. Amit bought two pencils and three chocolates for ₹11 and Sumeet bought one pencil and two chocolates for ₹7. Represent this situation in the form of a pair of linear equations. Find the price of one pencil and that of one chocolate graphically.

CASE STUDY (4 MARKS)

13. From Bangalore bus stand, if we buy 2 tickets to Malleshwaram and 3 tickets to Yeshwanthpur, the total cost is Rs. 46; but if we buy 3 tickets to Malleshwaram and 5 tickets to Yeshwanthpur, then the total cost is Rs. 74. Find the fares from Bangalore bus stand to Malleshwaram and Yeshwanthpur.

i) Represent above situation algebraically.

ii) What is the fare from Bangalore to Malleshwaram?

iii) From Bangalore, if Mini buys 4 tickets to Malleshwaram & 2 tickets to Yeshwanthpur what will be the total cost?

(1+2+1)

QUADRATIC EQUATIONS

IMPORTANT POINTS:

A polynomial of the form ax^2+bx+c, where a, b and c are real numbers and $a\neq0$ is called a quadratic polynomial.

Quadratic Equation

When we equate a quadratic polynomial to a constant, we get a quadratic equation.

Any equation of the form $p(x) = k$, where $p(x)$ is a polynomial of degree 2 and k is a constant, is a quadratic equation.

The standard form of a Quadratic Equation

The standard form of a quadratic equation is

$ax^2+bx+c=0$, where a,b and c are real numbers and $a\neq0$.

'a' is the coefficient of x^2. It is called the quadratic coefficient. 'b' is the coefficient of x. It is called the linear coefficient. 'c' is the constant term.

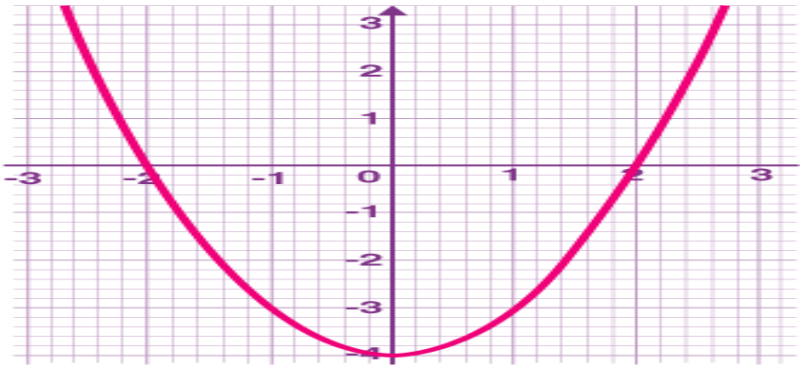
Solving Quadratic Equations by Factorisation Method to find the Roots of a Quadratic equation

The values of x for which a quadratic equation is satisfied are called the roots of the quadratic equation.

If α is a root of the quadratic equation $ax^2+bx+c=0$, then $a\alpha^2+b\alpha+c=0$.

A quadratic equation can have two distinct real roots, two equal roots or real roots may not exist.

Graphically, the roots of a quadratic equation are the points where the graph of the quadratic polynomial cuts the x-axis. Consider the graph of a quadratic equation $x^2-4=0$



Graph of a Quadratic Equation

In the above figure, -2 and 2 are the roots of the quadratic equation $x^2-4=0$

Note:

- If the graph of the quadratic polynomial cuts the x-axis at two distinct points, then it has real and distinct roots.
- If the graph of the quadratic polynomial touches the x-axis, then it has real and equal roots.
- If the graph of the quadratic polynomial does not cut or touch the x-axis then it does not have any real roots.

Solving a Quadratic Equation by Factorization method Consider a quadratic equation

$2x^2-5x+3=0$

$\Rightarrow 2x^2-2x-3x+3=0$

This step is splitting the middle term

We split the middle term by finding two numbers (-2 and -3) such that their sum is equal to the coefficient of x and their product is equal to the product of the coefficient of x^2 and the constant.

$(-2) + (-3) = (-5)$

MCQs (1 Mark Each)

Q.1 The general form of a quadratic equation is:

a) $ax^3+bx+c=0$ b) $ax+bx+c=0$ c) ax^2+bx+c d) $ax^2+bx+c=0$

Q.2 The quadratic equation has degree ……..

a) 0 b) 1 c) 2 d) 3

Q.3 A quadratic equation $ax^2+bx+c = 0$, has two equal roots if:

a) $b^2-4ac < 0$ b) $b^2-4ac > 0$ c) $b^2-4ac = 0$ d) none of these

Q.4 The sum of two numbers is 27 and product is 182. The numbers are:

a) 16 & 11 b) 13 & 14 c) 12 & 15 d) 3 & 24

Q.5 The maximum number of roots for a quadratic equation is ……

a) 1 b) 2 c) 3 d) 4

Q.6 The product of two consecutive positive integers is 110. To find the integers, this can be represented in the form of quadratic equation as:

a) $x^2+x+110 = 0$ b) $x^2+x-110 = 0$ c) $2x^2+x+110 = 0$ d) $x^2-x-110 = 0$

Q.7 The sum of the squares of two consecutive natural numbers is 313. The numbers are:

a) 12 & 13 b) 13 & 14 c) 11 & 12 d) 14 & 15

Q.8 The roots of the quadratic equation x^2-0.04 = 0 are:

a) $\pm$0.2 b) $\pm$0.02 c) 0.4 d) 2

Q.9 If one root of quadratic equation $ax^2+bx+c = 0$ is the reciprocal of the other, then:

a) b = c b) a = b c) ac = 1 d) a = c

Q.10 If $\frac{1}{2}$ is a root of the equation $x^2 - kx - \frac{5}{4} = 0$, then the value of k is:

a) 2 b) -2 c) 3 d) 1

Q.11 The linear factors of the quadratic equation $x^2+kx+1 = = 0$ ae:

a)$k \geq 2$ b) $k \leq 2$ c) $k \geq -2$ d) $2 \leq k \leq -2$

Q.12 If $x^2+y^2 = 25$ and $xy = 12$, then x is:

a) 3, 4 b) 3, -3 c) 3,4,-3 & -4 d) 3,

Q.13 The quadratic equation $x^2+x-5 = 0$ has:

a)two distinct real roots b) no real root

c) two equal real roots d) more than two real roots

Q.14 Which of the following equations has 2 as a root?

a) $x^2-4x+5 = 0$ b) $x^2+3x-12 = 0$ c) $2x^2-7x+6 = 0$ d) $3x^2-6x-2 = 0$

Q.15 If the equation $(m^2+n^2)x^2-2(mp+nq)x+p^2+q^2 = 0$ has equal roots, then:

b) $mp = nq$ b) $mq = np$ c) $mn = pq$ d) $mq = \sqrt{np}$

Q. 16)

Assertion : $4x^2-12x+9 = 0$ has repeated roots.

Reason : The quadratic equation $ax^2+bx+c = 0$ have repeated roots if discriminant $D > 0$.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q.17)

Assertion : The equation $x^2+3x+1 = (x-2)^2$ is a quadratic equation.

Reason : Any equation of the form $ax^2+bx+c = 0$ where $a \neq 0$, is called a quadratic equation.

(a) Both assertion (A) and reason ® are true and reason ® is the correct explanation of assertion (A).

(b) Both assertion (A) and reason ® are true but reason ® is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason ® is false.

(d) Assertion (A) is false but reason ® is true.

Q. 18

Assertion : The values of x are $-\frac{a}{}$, a for a quadratic equation $2x^2+ax-a^2 = 0$.

Reason : For quadratic equation $ax^2+bx+c = 0$ where $a \neq 0$, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q. 19

Assertion : The equation $8x^2+3kx+2 = 0$ has equal roots then the value of k is $\pm$ 8.

Reason : The equation $ax^2+bx+c = 0$ has equal roots if $D = b^2-4ac = 0$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Q. 20 **Assertion :** The roots of the quadratic equation $x^2+2x+2=0$ are imaginary.

Reason : If discriminant $D = b^2-4ac < 0$ then the roots of quadratic equation $ax^2+bx+c= 0$ are imaginary.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

VSA TYPE QUESTIONS (2 Marks Each)

Q.1 Find discriminant of the quadratic equation $3x^2+4x-5 = 0$. What type of roots does the given quadratic equation have?

Q.2 Find the roots of the quadratic equation $x^2-5x+6 = 0$.

Q.3 Find the value of p so that the quadratic equation $px(x-3)+9 = 0$ has two equal roots.

Q.4 A two digit number is four times the sum of the digits. It is also equal to 3 times the product of digits. Find the number.

Q.5 If one of the roots of $x^2+px-4 = 0$ is – 4, then find the value of p and product of its roots .

Q.6 If 1 is a root of the equations $ay^2+ay+3 = 0$ and $y^2+y+b = 0$, then find the value of ab.

SA TYPE QUESTIONS (3 Marks Each)

1. If $\sin\theta$ and $\cos\theta$ are roots of the equation $ax^2+bx+c = 0$, prove that $a^2-b^2+2ac = 0$.

2. Find the value of p, for which one root of the quadratic equation $px^2-14x+8 = 0$ is 6 times the other.

3. The sides of two square plots are $(2x-1)$m and $(5x+4)$m. The area of the second square plot is 9 times the area of the first square plot. Find the side of the larger plot.

4. Find the values of k for which the quadratic equation $(k+4)x^2+(k+1)x+1=0$ has equal roots.

5. The difference of two natural numbers is 5 and the sum of their reciprocal is $\frac{1}{10}$. Find the number.

6. Find the roots of the quadratic equation $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$.

LA TYPE QUESTIONS (5 Marks Each)

Q.1 A train covers a distance of 90 km at a uniform speed. Had the speed been 15 km/hr more, it would have taken 30 minutes less for the journey. Find the original speed of the train.

Q.2 The sum of the ages of Ram and his son Shyam is 45 years. Five years ago, the product of their ages in years was four times the Ram's age at that time. Find their present ages.

Q.3 Rajib takes 6 days less than the time taken by Mahid to finish a piece of work. If both Rajib and Mahid together can finish it in 4 days, find the time taken by Mahid to finish the work.

ANSWERS

CH. 4 QUADRATIC EQUAITONS

MCQs (1 MARK)									
1(D)	2(C)	3(C)	4(B)	5(B)	6(C)	7(A)	8(A)	9(D)	10(A)
11(D)	12(C)	13(A)	14(C)	15(B)	16(C)	17(D)	18(D)	19(A)	20(A)

VSA TYPE QUESTIONS (2 MARKS)		
1. D = 76, Two distinct real roots	2. 2,3	3. P = 4
4. 24	5. P = 3, Product of roots is -4	6. ab = 3

SA TYPE QUESTIONS (3 MARKS)		
1. TO PROVE	5. P = 3	2. 39 cm
3. K = -3, 5	7. 5, 10	6. x = $-52\sqrt{2}, -\sqrt{2}$

LA TYPE QUESTIONS (5 MARKS)		
1. 45 km/hr	2. Ram's age = 36 years Shyam's age = 9 years	3. 12 Days

ARITHMETIC PROGRESSION

IMPORTANT POINTS:

❖ An arithmetic progression (AP) is a list of numbers in which each term is obtained by adding a fixed number d to the preceding term, except the first term. The fixed number d is called the common difference.

The general form of an AP is a, a + d, a + 2d, a + 3d, ……..

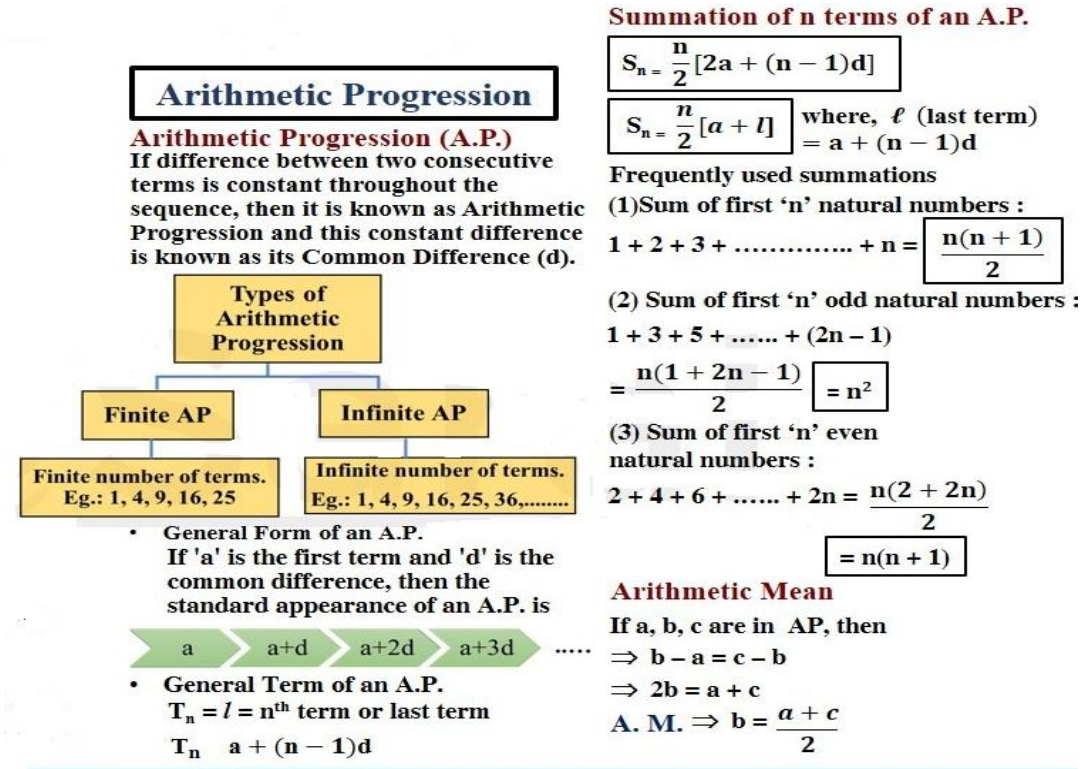
❖ A given list of numbers a_1 , a_2 , a_3, a_4, a_5, . . . is an AP, if the differences $a_2 - a_1$, $a_3 - a_2$, $a_4 - a_3$, . . ., give the same value, i.e., $a_{k+1} - a_k$ is the same for different values of k.

❖ In an AP with first term a and common difference d, the nth term (or the general term) is given by $a_n = a + (n - 1) d$.

❖ The sum of the first n terms of an AP is given by :$S_n = \frac{n}{2}[2a + (n-1)d]$

❖ If l is the last term of the finite AP, say the nth term, then the sum of all terms of the AP is given by : $S_n = \frac{n}{2}(a + l))$

❖ If a, b, c are in AP, then b = $\frac{a+c}{2}$ and b is called the arithmetic mean of a and c



VSA (1)

Q1. 20th term of the A.P: 10, 7, 4, ..., is

(a) 67 (b) 47 (c) -47 (d) -77

Answer: (c) -47

Explanation: Given, A.P. = 10, 7, 4, ...

First term, $a = 10$, Common difference, $d = a_2 - a_1 = 7-10 = -3$

As we know, for an A.P., $a_n = a + (n-1)d$

Putting the values;

$A_{20} = 10+(20-1)(-3)$

$A_{20} = 10+(19)(-3)$

$A_{20} = 10-57 = -47$

Q2. In an AP, if $d = -4$, $n = 7$, $a_n = 4$, then a is

(a) 6 (b) 7 (c) 20 (d) 28

Answer: (d) 28

Solution; Given, $d = -4$, $n = 7$, $a_n = 4$

We know that, $a_n = a + (n - 1)d$

$4 = a + (7 - 1)(-4)$

$4 = a + 6(-4)$

$4 = a - 24$

$\Rightarrow a = 4 + 24 = 28$

Q3. The missing terms in AP: __, 13, __, 3 are:

(a) 11 and 9 (b) 17 and 9 (c) 18 and 8 (d) 18 and 9

Answer: **(c)**

Explanation: $a_2 = 13$ and $a_4 = 3$

The nth term of an AP;

$a_n = a+(n-1)\,d$

$a_2 = a +(2-1)d$

$13 = a+d$ ..... (i)

$a_4 = a+(4-1)d$

$3 = a+3d$ ..... (ii)

Subtracting equation (i) from (ii), we get,

$-10 = 2d$

$d = -5$

$13 = a+(-5)$

$a_3 = 18+(3-1)(-5)= 18+2(-5) = 18-10 = 8$ (third term).

Q4. **The sum of the first five multiples of 5 is:**

(a) 75 (b) 85 (c) 65 (d) -75

Answer: **(a) 45**

Explanation: The first five multiples of 5 is 5, 10, 15, 20 and 25

a=5 and d=5 n=5

Sum, $S_n = n/2[2a+(n-1)d]$

$S_5 = 5/2[2(5)+(5-1)5]=5/2[10+20]=5/2[30]=5 \times 15= 75$

Q5. **Statement -1 (A): The sum of 15 terms of the A.P. 1,3,5,7,……. is 225.**

Statement -2 (R): The sum of first n odd natural numbers is n^2

(a) Both Assertion (A)and Reason (R)are true and Reason (R) is the correct explanation of Assertion(A).

(b) Both Assertion(A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion(A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Answer : (a)

1,3,5,7,……, (2n-1) are first n odd natural numbers. Let S be their sum. Then

$S= 1+3+5+7+\ldots\ldots\ldots+(2n-1)$

$S=n/2(1+2n-1) = n^2$

So, statement 2 is true. Using this statement , we find that

$1+3+5+7+\ldots\ldots$ up to 15 terms $= 15^2 =225$

So, statement-1 is also true and ststement-2 is a correct explanation for statement -1.

For practice :

1. The 15^{th} term from the last of the AP 7, 10, 13,…, 130 is
   a. 49 b.85 c. 88 d.110 (1 mark)
2. If k-1, k+1 and 2k+3 are in AP, then the value of k is
   a. -2 b.0 c.2 d. 4 (1 mark)
3. If the 2^{nd} term of an AP is 13 and the 5^{th} term is 25, then its 7^{th} term is
   a. 30 b. 33 c.37 d.38 (1 mark)

4. If the last term of the AP 5, 3, 1, -1, …… is -41, then the AP consists of

a. 46 terms b. 25 terms c.24 terms d.23 terms

5. Statement -1 (A): a,b,c are in A.P. if and only if 2b=a+c

Statement -2 (R): The sum of first n odd natural numbers is n^2.

(a) Both Assertion (A)and Reason (R)are true and Reason (R) is the correct explanation of Assertion(A).

(b) Both Assertion(A) and Reason(R) are true but Reason(R) is not the correct explanation of Assertion(A).

(c) Assertion (A) is true but Reason (R) is false.

(d) Assertion (A) is false but Reason (R) is true.

Answer: 1(a) 2(b) 3(c) 4(d) 5(b)

SA(2)

Q1. How many multiples of 7 lie between 20 to 300 ?

Answer:

The multiples of 7 lying between 20 to 300 are 21,28,35,……., 294 in A.P. with

First term= 21

Common difference ,d=7

and last term a_n=294

Let n multiples of 7 lie between 20 to 300

a_n =a+(n-1)d

or 294 = 21 +(n-1)x7

or 294 = 21 +7n-7

or 7n = 280

or n = 40 (answer)

Q.2. Find the ratio of 10^{th} term from beginning and 20^{th} term from the end of the A.P. : 3,8,13,…..,253

Answer :

We have 3,8,13,……., 253 an A.P.

Here a = 3 , d = 8-3 = 5, l= 253

10^{th} term from beginning , a_{10} = a +(10-1)d

= 3+9x5

= 48

20^{th} term from the end , a_{20} = l-(20-1)d

= 253-19x5

= 158

Therefore ratio of a_{10} and a_{20} = 48/158= 24/79

i.e. $a_{10} : a_{20} = 24 : 79$

Q3. Find the sum of first n-terms of an A.P. whose nth term is 3n+7.

Answer: $a_n = 3n+7$ (given)

First term $a_1 = 3x1+7 = 10$

Second term $a_2 = 3x2+7 = 13$

Common difference = d = 13-10 =3

Sum of first n terms of an A.P. is given by

❖ $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 10 + (n-1)3]$

$= \frac{n}{2}[20 + 3n - 3]$

$S_n = \frac{n}{2}[17 + 3n]$ (Answer)

For practice :

Q1. The sum of 4th and 8th terms of an AP is 24 and the sum of the 6th and 10th terms is 44. Find the first three terms of the AP.

Q2. How many number of multiples of 4 lie between 10 and 250?

Q3. . Find the sum of the first 22 terms of an AP in which d = 7 and 22nd term is 149.

Answer 1. The first three terms of this AP are −13, −8, and −3.

2. 9. n = 60,

3. 1661

SA(3)

Q1. If a,b , c are in A.P. , then show $\frac{1}{bc}$, $\frac{1}{ca}, \frac{1}{ab}$ are also in A.P.

Answer : $\frac{1}{bc}$, $\frac{1}{ca}, \frac{1}{ab}$ will be in A.P. if and only if

$$\frac{2}{ca} = \frac{1}{bc} + \frac{1}{ab}$$

RHS $= \frac{1}{bc} + \frac{1}{ab}$

$= \frac{a+c}{abc} = \frac{2b}{abc} = \frac{2}{ac} = LHS \quad [As\ a, b, c\ are\ in\ A.P. \quad \therefore \quad a + c = 2b]$

Hence $\frac{1}{bc}$, $\frac{1}{ca}, \frac{1}{ab}$ are in A.P.

Q2. If $S_n = 3n^2 - 4n$, then find the nth term.

Answer :

$S_n = 3n^2 - 4n$

If n=1 , $S_1 = 3x1^2-4x1 = -1$

n =2, $S_2 = 3x2^2-4x2 = 12-8 = 4$

n=3 $S_3 = 3x3^2-4x3 = 27-12 = 15$

n=4 $S_3 = 3x4^2-4x4 = 48-16 = 32$

second term $a_2 = S_2 - S_1 = 4+1 = 5$

third term $a_3 = S_3 - S_2 = 15- 4 = 11$

common difference d= $a_3 - a_2 = 11 - 5 = 6$

let a be the first term

$a_2 = a + d$

$5 = a + 6$

$a = -1$

nth term $a_n = a +(n-1)d$

$= -1 +(n-1)6$

$a_n = 6n -7$

Q3. In an A.P., sum of first ten terms is -150 and the sum of its next ten terms is -550. Find the A.P.

Answer : let a be the first term and d be the common difference of the A.P.

sum of first 10 terms, $S_{10} = \frac{10}{2}[2a + 9d]$

$= 5(2a + 9d)$

$-150 = 5(2a + 9d)$

$2a + 9d = -30$ .....(i)

Sum of next 10 terms , $S_{n10} = \frac{10}{2}(2a_{11} + 9d)$

$-550 = 5(2(a + 10d) + 9d)$

$-110 = 2a + 20d + 9d$

$2a + 29d = -110$ .....(ii)

subtracting (i) from (ii), we get

$20d = -80$

$d = -4$

putting d= -4 in (i) we get

$2a + 9(-4) = -30$

$a = 3$

Therefore A.P. is 3, (3-4), (3-8),……. i.e. 3,-1,-5,……

For practice :

1. How many terms of the AP: 24, 21, 18, . . . must be taken so that their sum is 78?
2. The sum of the third and the seventh terms of an AP is 6 and their product is 8. Find the sum of the first sixteen terms of the AP.
3. The first term of an AP is 5, the last term is 45 and the sum is 400. Find the number of terms and the common difference

Answers : 1. 4 or 13. Both values of n are admissible.

So, the number of terms is either 4 or 13.

2. . $S_{16} = 20$

3. Number of terms, n = 16, Common difference d = 40/15 = 8/3

LA(5)

Q1. The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and last term to the product of the middle terms is 7: 15. Find the number.

Answer : Let the four consecutive terms of A.P. be (a - 3d), (a-d) , (a+ d) and (a + 3d)

By given conditions

$$a - 3d + a - d + a + d + a + 3d = 32$$

Or $4a = 32$

Or $a = 8$

And $\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$

$d^2 = 4$

$d = \pm 2$

Hence the numbers are 2, 6, 10, 14 or 14, 10, 6, 2

For practice

Q2. The first term of an A.P. is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and common difference of the A.P.

Answer : n = 21 and d= 4

Case study(4)

Q1. There are 25 trees at equal distances of 5m in a line with a well, the distance of the well from the nearest tree being 10m. A gardener waters all the trees separately starting from the well and he returns to the well after watering each tree to get water for the next.

(a) Find how much distance the gardener has to cover in order to water first tree.

(b) Find how much distance the gardener has to cover in order to water second tree.

(c) Find how much distance the gardener has to cover in order to water 25^{th} tree.

(d) Find the total distance the gardener will cover in order to water all the trees.

Answer

For first tree =20 m distance is covered in to fro from well

For second tree =2(10+1×5) =30 m distance is covered in to fro from well

For third tree =2(10+2×5) =40 m distance is covered in to fro from well

....

For 25th tree = 2(10+24×5)=260 m distance is covered in to-fro from well

So total distance

=20 + 30 + 40 +....+ 260

This is a A.P with a=20,d=10 and n=25

$$S_n = \frac{n}{2}[2a + (n-1)d]$$
$$S_{25} = \frac{25}{2}[2\text{x}20 + (25-1)10]$$
$$S_{25} = \frac{25}{2}[40 + 24 \times 10]$$
$$S_{25} = 25[20 + 12 \times 10]$$
$$S_{25} = 25\text{x}140$$
$$S_{25} = 3500\text{m}$$

Q2. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performances. If each price is Rs 20 less than its preceding prize.

(i) Find the value of first prize.
(ii) Find the value of second prize.
(iii) Find the value of third prize.
(iv) Find the value of last prize.

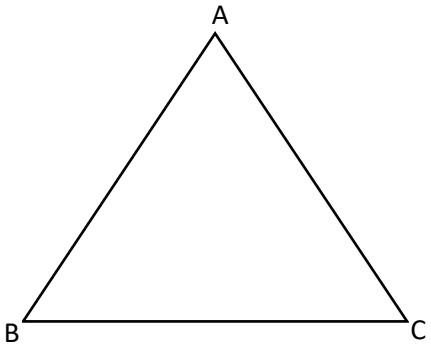
Answer : (i) Rs. 160 (ii) Rs. 140 (iii) Rs. 120 (iv) Rs. 40

TRIANGLES

IMPORTANT POINTS:

PREVIOUS KNOWLEDGE:

- Polygons- A polygon is a geometrical 2-D (plane figure) shape that is formed with straight lines.
  e.g. - if it is formed by three straight lines is called triangle.

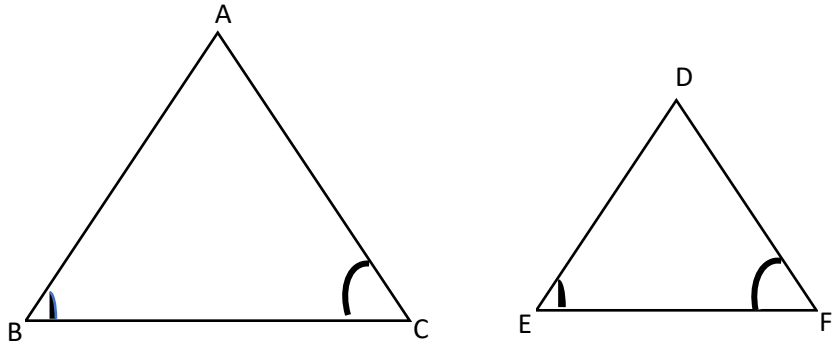


From the above figure ABC is a triangle

- Where $\angle A + \angle B + \angle C = 180$

SIMILARITY OF POLYGONS:

Two polygons of the same number of sides are similar, if (i) their corresponding angles are equal and (ii) their corresponding sides are in the same ratio (i.e. proportional)



From the above figures triangles ABC and DEF are similar to each other.

Therefore, we can conclude that two similar figures have the same shape but not necessarily the same size.

CRITERIA FOR SIMILARITY OF TRIANGLES:

In △ABC and △DEF

(i) $\angle A=\angle B$, $\angle B=\angle E$, $\angle C=\angle F$ and

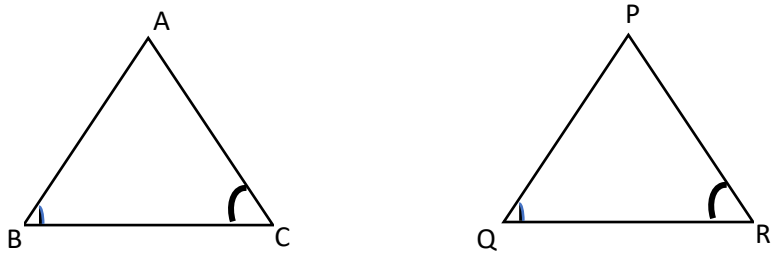
(ii) $\frac{AB}{DE}=\frac{BC}{EF}=\frac{CA}{FD}$

(i) Angle- Angle-Angle (AAA) Similarity Criterion: In two triangles, if corresponding angles are equal, then the triangles are similar.

Therefore, two similar triangles satisfy AAA criteria.

Hence $\triangle$ABC~ $\triangle$DEF

REMARK: If two angles of a triangle are respectively equal to the two angles of another triangle, then the angle sum property of a triangle their third angles will also be equal. Therefore, AAA similarity can also be stated



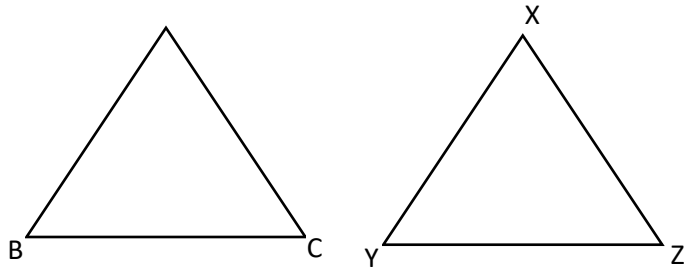
In $\triangle$ABC and $\triangle$PQR if corresponding two angles are equal i.e. $\angle B=\angle Q$ and $\angle C=\angle R$

Also, we know, the sum of all angles of a triangle is $180°$

$\angle A+\angle B+\angle C=180°=\angle P+\angle Q+\angle R$

1. $\angle A=\angle P$

Therefore, equality of two corresponding angles of two triangles (AA-Criterion) implies equality of theirs



third angle.

(ii) Side-Side-Side (SSS) Similarity Criterion: In two triangles if the sides of one triangle are proportional to the sides of another triangle, then two triangles are similar and hence corresponding angles are equal.

If $\frac{AB}{XY}=\frac{BC}{YZ}=\frac{CA}{ZX}$

$\therefore$ $\triangle$ABC~ $\triangle$XYZ

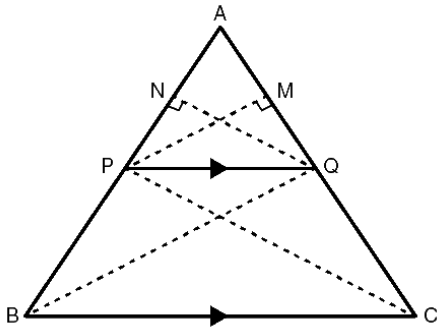
then $\angle A=\angle X$, $\angle B=\angle Y$ and $\angle C=\angle Z$

(iii) Side-Angle-Side (SAS) Similarity Criterion: If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then two triangles are similar.

If $\frac{AB}{XY}=\frac{AC}{XZ}$ and $\angle A=\angle X$, then $\triangle$ABC ~$\triangle$ XYZ

Basic Proportionality Theorem (B.P.T.) (Thales Theorem):

In a triangle, a line drawn parallel to one side, to intersect the other sides in distinct points, divides the two sides in the same ratio.



Given: In $\triangle$ABC, PQ is parallel to BC, i.e. PQ$\parallel$BC

To be proved: $\frac{AP}{PB}=\frac{AQ}{QC}$

Construction: Join BQ and PC and then draw QN$\perp$AB, PM$\perp$AC

Proof: The area of $\triangle$APQ=1/2 × Base ×Height

$= 1/2\times AP\times QN$

Similarly, the area of $\triangle$PBQ= $1/2\times PB\times QN$

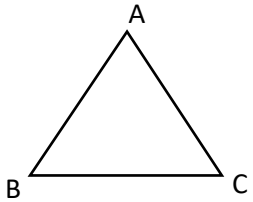
Area of $\triangle$APQ=$1/2\times AQ\times PM$

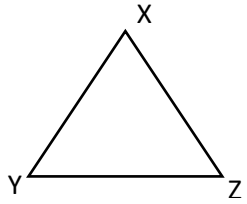
Also, area of $\triangle$QCP=$1/2\times QC\times PM$

Now, $\frac{Area\ of\ \triangle\ APQ}{Area\ of\ \triangle\ PBQ}=\frac{\frac{1}{2}\times AP\times QN}{\frac{1}{2}\times PB\times QN}=\frac{AP}{PB}$ ..... (i)

Similarly,

$\frac{Area\ of\ \triangle\ APQ}{Area\ of\ \triangle\ QCP}=\frac{\frac{1}{2}\times AQ\times PM}{\frac{1}{2}\times QC\times PM}=\frac{AQ}{QC}$ ..... (ii)





We know that the triangles with same parallel lines and on the same base have equal areas

Therefore, from (i) and (ii) we have

(i)=(ii)

2. $\frac{AP}{PB}=\frac{AQ}{QC}$

Hence, the basic proportionality theorem is proved.

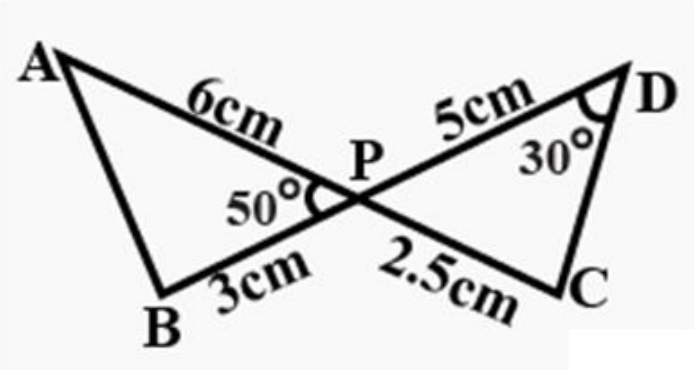
Converse of Basic Proportionality Theorem:

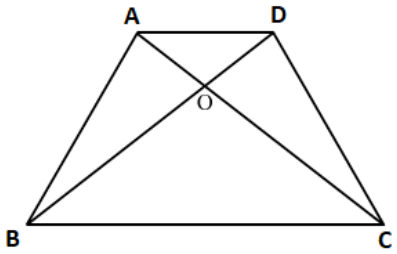
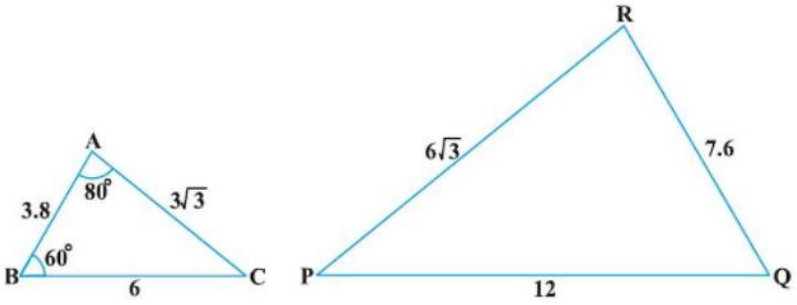
According to this theorem, if a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (Proof-Do it yourself)

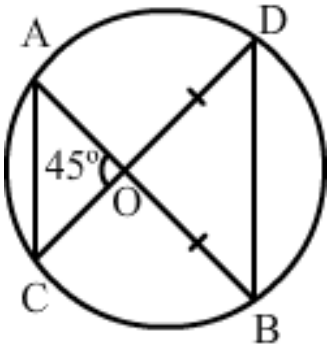
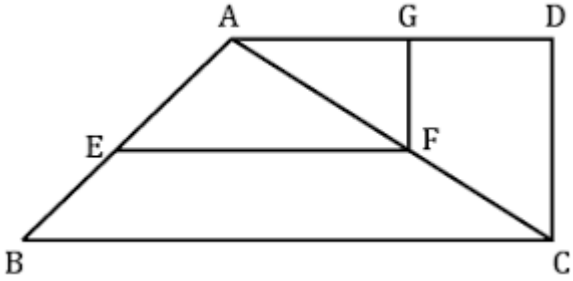
From above triangle we have to remind,

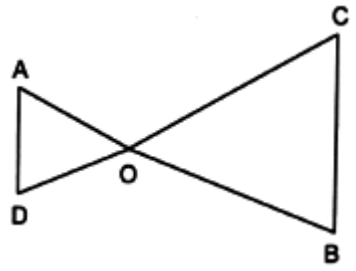
I) $\frac{AP}{PB}=\frac{AQ}{QC}$ ii)$\frac{AB}{AP}=\frac{AC}{AQ}$ iii) $\frac{AB}{PB}=\frac{AC}{QC}$

Q. NO	QUESTION	MARK
1	D and E are respectively the points on the sides AB and AC of a triangle ABC such that AD = 2 cm, BD = 3 cm, BC = 7.5 cm and DE||BC. Then, length of DE (in cm) is (a) 2.5 (b) 3 (c) 5 (d) 6	1
2	If Δ ABC ~ Δ EDF and Δ ABC is not similar to ΔDEF, then which of the following is not true? (a) BC . EF = A C. FD (b) AB . EF = AC . DE (c) BC . DE = AB . EF (d) BC . DE = AB . FD	1
3	If in two triangles ABC and PQR, $\frac{AB}{QR}=\frac{BC}{PR}=\frac{CA}{PQ}$, then (a) Δ PQR ~ Δ CAB (b) Δ PQR ~ Δ ABC (c) Δ CBA ~ Δ PQR (d) Δ BCA ~ Δ PQR	1
4	In the given Fig., two line segments AC and BD intersect each other at the point P such that PA = 6 cm, PB = 3 cm, PC = 2.5 cm, PD = 5 cm, ∠APB = 50° and∠CDP = 30°. Then, ∠PBA is equal to	1

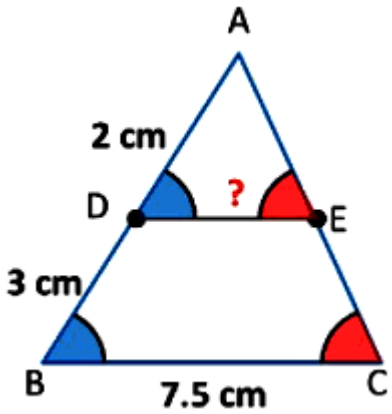
	 (a) 50° (b) 30° (c) 60° (d) 100°	
5	. If in two triangles DEF and PQR, ∠D =∠Q and ∠R = ∠E, then which of the following is not true? (a) $\frac{EF}{PR} = \frac{DF}{PQ}$ (b) $\frac{DE}{PQ} = \frac{EF}{RP}$ (c) $\frac{DE}{QR} = \frac{DF}{PQ}$ (d) $\frac{EF}{RP} = \frac{DE}{QR}$	1
6	In triangles ABC and DEF, ∠B = ∠E, ∠F = ∠C and AB = 3 DE. Then, the two triangles are (a) Congruent but not similar (b) similar but not congruent (c) neither congruent nor similar (d) congruent as well as similar	1
7	If in triangles ABC and EDF, $\frac{AB}{DE} = \frac{BC}{FD}$, then they will be similar, when (a) ∠B = ∠E (b) ∠A = ∠D (c) ∠B = ∠D (d) ∠A = ∠F	1
8	It is given that Δ ABC ~ ΔDFE, ∠A =30°,∠C = 50°, AB = 5 cm, AC = 8 cm and DF= 7.5 cm. Then, the following is true: (a) DE = 12 cm, ∠F = 50° (b) DE = 12 cm, ∠F = 100° (c) EF = 12 cm, ∠D = 100° (d) EF = 12 cm, ∠D = 30°	1
9	A vertical pole 6 m long casts a shadow of length 3.6 m on the ground. What is the height of a tower which casts a shadow of length 18 m at the same time? (a) 10.8 m (b) 28.8 m (c) 32.4 m (d) 30 m	1
10	In Δ ABC, DE‖ BC so that AD = 2.4 cm, AE = 3.2 cm and EC = 4.8 cm. Then AB = ? (a) 3.6 cm (b) 6 cm (c) 6.4 cm (d) 7.2 cm	1

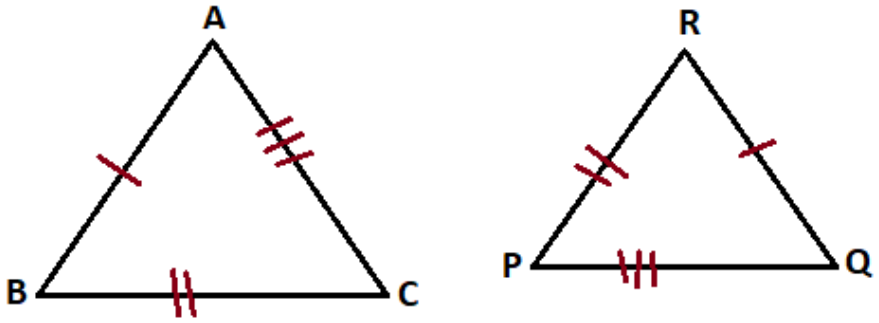
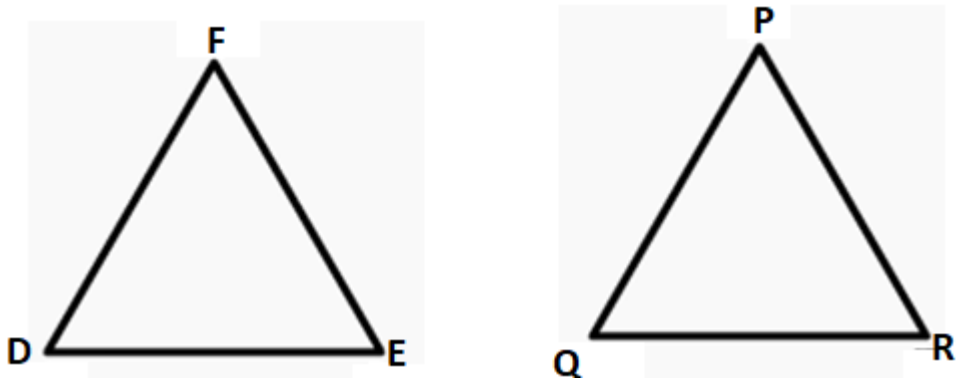
11	In a Δ ABC, DE is drawn parallel to BC, cutting AB and AC at D and E respectively such that AB = 7.2 cm, AC = 6.4 cm and AD = 4.5 cm. Then AE = ? (a) 5.4 cm (b) 4 cm (c) 3.6 cm (d) 3.2 cm	1
12	In Δ ABC, DE‖ BC such that $\frac{AD}{DB} = \frac{3}{5}$. If AC = 5.6 cm, then AE = ? (a) 4.2 cm (b) 3.1 cn (c) 2.8 cm (d) 2.1 cm	1
13	ΔABC ~ ΔDEF and the perimeters of Δ ABC and ΔDEF are 30 cm and 18 cm respectively. If BC = 9 cm, then EF = ? (a) 6.3 cm (b) 5.4 cm (c) 7.2 cm (d) 4.5 cm	1
14	ABCD is a trapezium such that BC ‖ AD and AD = 4 cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$ then BC =  (a) 7 cm (b) 8 cm (c) 9 cm (d) 6 cm	1
15	Find ∠P in the adjoining figure.  (a) 40° (b) 80° (c) 60° (d) None of these	1

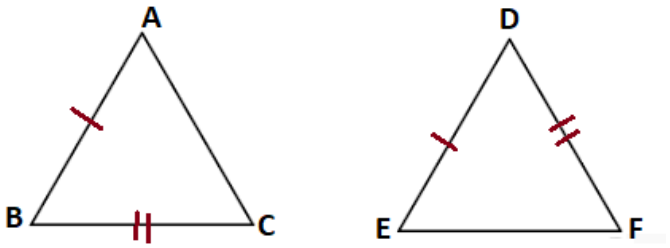
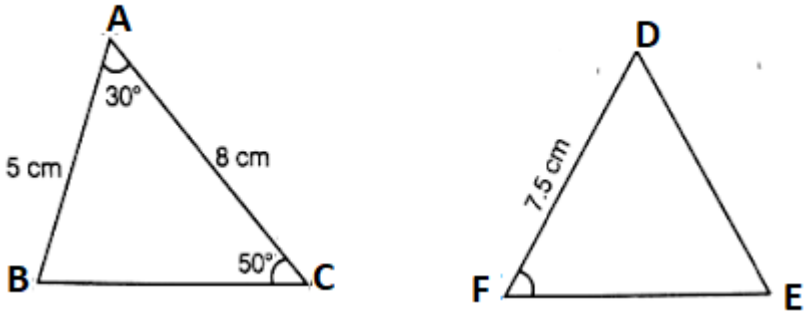
16	In the given figure, O is the point of intersection of two chords AB and CD such that OB = OD and ∠AOC = 45°. Then, ΔOAC and ΔODB are (a) equilateral and similar (b) equilateral but not similar (c) isosceles and similar (d) isosceles but not similar 	1
17	Assertion (A):- All rhombus are similar to each other. Reason (R):- Two same polygons are similar it their corresponding angles are equal and the lengths of their corresponding sides are proportional. (a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A). (c) Assertion (A) is true but reason (R) is false. (d) Assertion (A) is false but reason (R) is true.	1
18	Assertion (A): If BC ||EF and FG||CD then, $\frac{AE}{AB} = \frac{AG}{AD}$  Reason (R): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. (a) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true, but reason (R) is not the correct explanation of assertion (A). (c) Assertion (A) is true, but reason (R) is false. (d) Assertion (A) is false, but reason (R) is true.	1
19	Assertion (A): In ΔABC & Δ DEF , $\frac{AB}{DE} = \frac{BC}{EF}$ *and* ∠B= ∠D, then $\Delta ABC \sim$ Δ DEF. Reason (R) :- If one angle of a triangle is equal to one angle of the other triangle	1

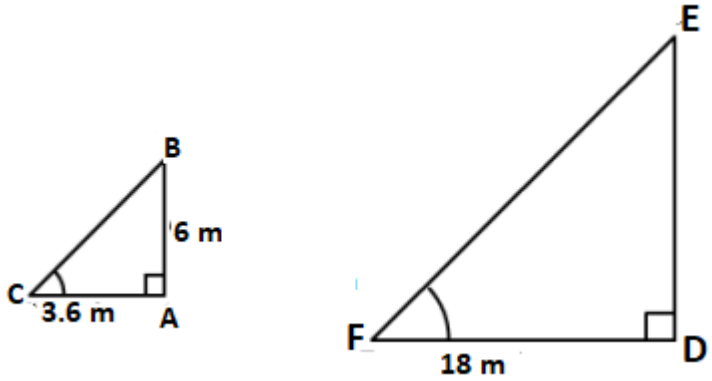
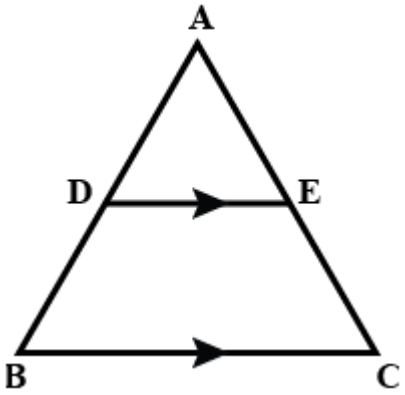
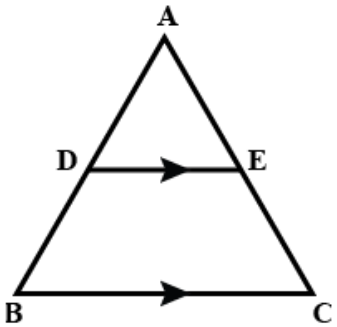
	and the sides including these angles are proportional, then two triangles are always similar. Reason (R): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. (a) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true, but reason (R) is not the correct explanation of assertion (A). (c) Assertion (A) is true, but reason (R) is false. (d) Assertion (A) is false, but reason (R) is true.	
20	Assertion (A): E and F are points on the sides PQ and PR respectively of a Δ PQR, such that PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm, then EF ∥ QR. Reason (R) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (a) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true, but reason (R) is not the correct explanation of assertion (A). (c) Assertion (A) is true, but reason (R) is false. (d) Assertion (A) is false, but reason (R) is true.	1
21	Assertion (A) :- In the given fig. OA × OB = OC × OD, then ∠A = ∠C and ∠B = ∠D.  Reason (R) :- If two triangles are similar, then their corresponding angles are equal. Reason (R) If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side. (a) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true, but reason (R) is not the correct explanation of assertion (A). (c) Assertion (A) is true, but reason (R) is false.	1

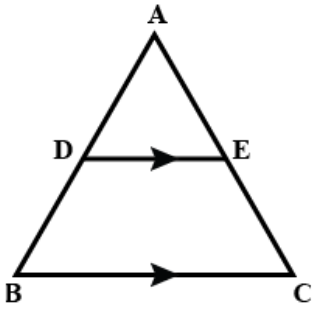
	(d) Assertion (A) is false, but reason (R) is true.	
22	Assertion (A): If ΔABC and ΔPQR are congruent triangles, then they are also similar triangles. Reason (R): All congruent triangles are similar but the similar triangles need not be congruent. (a) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A). (b) Both assertion (A) and reason (R) are true, but reason (R) is not the correct explanation of assertion (A). (c) Assertion (A) is true, but reason (R) is false. (d) Assertion (A) is false, but reason (R) is true.	1

Q. NO	**ANSWERS**
1	**Ans:- (c)** In ΔADC and ΔABC ∠ADE = ∠ABC (corresponding angles) ∠A = ∠A (Common) △ADE ~ △ABC (AA similarity) $\frac{AD}{DB} = \frac{DE}{BC}$ (CPST) $\therefore \frac{2}{3} = \frac{DE}{7.5}$ => DE = 5 cm 
2	**Ans:- (c)** Since, Δ ABC ~ Δ EDF $\therefore \frac{AB}{ED} = \frac{BC}{DF} = \frac{AC}{EF}$ (CPST) If we take $\frac{AB}{ED} = \frac{BC}{DF}$, then AB × DF = ED × BC, So (d) is true If we take $\frac{BC}{DF} = \frac{AC}{EF}$, then BC × EF = DF × AC, so (a) is true If we take $\frac{AB}{ED} = \frac{AC}{EF}$, then AB × EF = ED × AC, so (b) is true

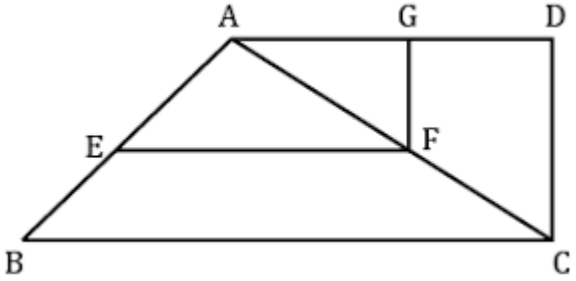
3	**Ans (a)**  By SSS criterion, the corresponding vertices are as follows: C ↔ P , A ↔ Q, B↔ R Therefore, △PQR ~ △CAB
4	**Ans:- (d)** Given: In Δ ABP and ΔCPD $\frac{PA}{PD} = \frac{6}{5}$, $\frac{PB}{PC} = \frac{3}{2.5} = \frac{6}{5}$ $\angle APB = \angle CPD = 50^o$ ∴ △APB ~△DPC (SAS similarity) ∠A=∠D=30^0 $\angle PBA = 180^o - (50^o + 30^o) = 100^o$
5	:- Ans:- (b)  In △DEF and △QRP, ∠D=∠Q, ∠R=∠E △DEF~△QRP (AA similarity) $\frac{DE}{QR} = \frac{EF}{RP} = \frac{DF}{PQ}$

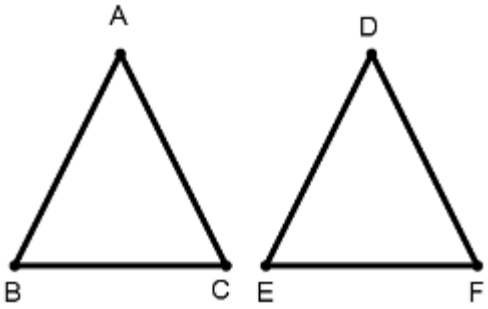
	Therefore, option (a), (c) and (d) are true but (b) is not true.
6	Ans:- (b) In △ABC and △DEF ∠B=∠E (Given) , ∠C=∠F (Given) ∴△ABC~△DEF (AA similarity) It is given that AB = 3DE It is clear that the sides AB and DE are not equal So, the triangles are not congruent. Therefore, the triangles ABC and DEF are similar but not congruent.
7	Ans:- (c)  In Δ ABC and ΔDEF, $\frac{AB}{DE} = \frac{BC}{FD}$ (Given) ∴ Δ ABC ~ ΔEDF , when ∠B = ∠D by using (SAS similarity)
8	Ans:- (b)  ∴ Δ ABC ~ ΔDFE Therefore, $\frac{AB}{DF} = \frac{BC}{EF} = \frac{AC}{DE}$ (CPST) $=> \frac{5}{7.5} = \frac{8}{DE}$ ∴ DE = 12 cm In Δ ABC, ∠A + ∠B + ∠C = 180° $=>$ 30° + ∠B + 50° = 180° ∴ ∠B = 100°

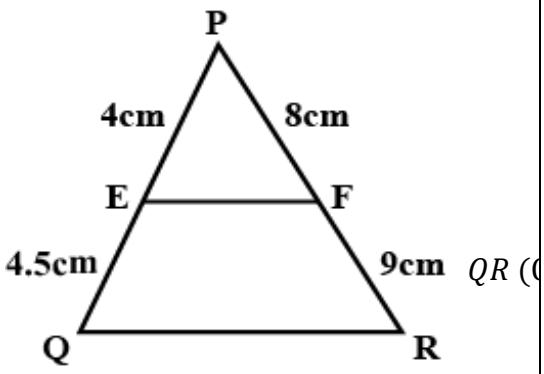
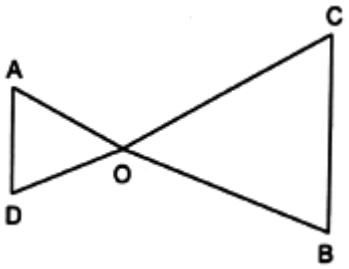
	$\therefore \Delta ABC \sim \Delta DFE$ $\angle A = \angle D = 30^o$, $\angle B = \angle F = 100^o$, $\angle C = \angle E = 50^o$ (CPST)
9	Ans:- (d)  In ΔABC & ΔDEF $\angle C = \angle F$ (Angular elevation of the sun at the same time) $\angle A = \angle D = 90^o$ $\therefore \Delta ABC \sim \Delta DEF$ $(AA\ similarity)$ Therefore, $\frac{AB}{DE} = \frac{AC}{DF}$ (CPST) $=> \frac{6}{DE} = \frac{3.6}{18}$ DE = 30 m
10	**Ans:- (b)** Since, DE ‖ BC, Therefore, $\frac{AD}{DB} = \frac{AE}{EC}$ (Thales theorem) $\frac{2.4}{DB} = \frac{3.2}{4.8}$ $=>$ DB = 3.6 cm. Therefore, AB = 2.4 + 3.6 = 6 cm 
11	**Ans:- (b)** Since, DE ‖ BC Therefore, $\frac{AD}{AB} = \frac{AE}{AC}$ (On the basis of B.P.T) $\frac{4.5}{7.2} = \frac{AE}{6.4}$ $=>$ AE = 4 cm. 

12	**Ans:- (d)** DE ∥ BC, Therefore, $\frac{AD}{DB} = \frac{AE}{EC}$ (Thales theorem) $\frac{3}{5} = \frac{x}{5.6-x}$ (let AE = x cm) $\Rightarrow$ 16.8 – 3x = 5x $\Rightarrow$ x = 2.1 cm 
13	**Ans:- (b)** ΔABC ~ ΔDEF Therefore, $\frac{P(\Delta ABC)}{P(\Delta DEF)} = \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{EF}$ $\Rightarrow \frac{P(\Delta ABC)}{P(\Delta DEF)} = \frac{BC}{EF}$ $\Rightarrow \frac{30}{18} = \frac{9}{EF}$ $\Rightarrow$ EF $= \frac{9\times18}{30} = 5.4\ cm$
14	**Ans:- (b)** In ΔAOD and ΔCOB, ∠OAD = ∠OCB [alternate angles] ∠ODA = ∠OBC [alternate angles] . ∴ ΔAOD ~ ΔCOB (AA – similarity) Therefore, $\frac{AO}{CO} = \frac{DO}{BO} = \frac{AD}{BC}$ (CPST) $\Rightarrow \frac{1}{2} = \frac{AD}{BC}$ $\Rightarrow \frac{1}{2} = \frac{4}{BC}$ Hence, BC = 8 cm
15	**Ans:- (a)** From the figure, $\frac{RQ}{AB} = \frac{PQ}{BC} = \frac{RP}{AB} = \frac{1}{2}$ Therefore, $\Delta ABC \sim \Delta RQP$ (SSS similarity) ∠P = ∠C = [180^o – (80^o + 60^o)] = 40^o
16	Ans (c) In ΔAOC and ΔODB, ∠A = ∠D & ∠C = ∠B (Angles in the same segment) Therefore ΔAOC ~ ΔODB (AA similarity) $\Rightarrow \frac{OC}{OB} = \frac{OA}{OD} = \frac{AC}{BD}$ (CPST)

	Now, OB= OD (Given) $\Rightarrow\ OC = OA$ Therefore, ΔAOC and ΔODB isosceles and similar
17	Ans:- (d) The ratio of sides of two rhombus is same but their angles may not be same. Therefore, Assertion is false.
18	Ans :- (a) In Δ $ABC, EF \parallel$ BC , $\frac{AE}{EB} = \frac{AF}{FC}$ (Thales Theorem) $\Rightarrow \frac{EB}{AE} = \frac{FC}{AF}$ (Taking reciprocal) $\Rightarrow \frac{EB}{AE} + 1 = \frac{FC}{AF} + 1$ $\Rightarrow \frac{EB + AE}{AE} = \frac{FC + AF}{AF}$ $\Rightarrow \frac{AB}{AE} = \frac{AC}{AF}$ $\Rightarrow \frac{AE}{AB} = \frac{AF}{AC}$ (Taking reciprocal) ---------- (i) Similarly, in $\Delta AFC,\ FG \parallel$ CD $\frac{AF}{FC} = \frac{AG}{AD}$ ---------- (ii) From (i) & (ii) $\frac{AE}{AB} = \frac{AG}{AD}$ Therefore, Assertion is correct and reason explain the assertion.
19	**Ans (d)** In ΔABC & Δ DEF , $\frac{AB}{DE} = \frac{BC}{EF}$ *and* ∠B= ∠D. It will be similar when ∠B = ∠E. So, ΔABC & Δ DEF are not similar. Therefore, Assertion (A) is false, But Reason is true.





20	Solution:- Ans(a) In the given fig. $\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$ $\frac{PF}{FR} = \frac{8}{9}$ Since, $\frac{PE}{EQ} = \frac{PF}{FR}$ Therefore, EF ∥ QR (C Therefore, Assertion (A) is true. Reason (R) is also correct and it explains the reason. So, option (A) is true. 
21	Ans :- (a)  Since, OA × OB = OC × OD Therefore, $\frac{OA}{OC} = \frac{OD}{OB}$ ---------- (i) Now, in Δ AOD & Δ COB $\frac{OA}{OC} = \frac{OD}{OB}$ (from eqn. (i) Also, ∠AOD = ∠COB (Vertically opposite angles) Therefore, Δ AOD ~ Δ COB (SAS similarity) ∠A = ∠C and ∠D = ∠B (CPST) Therefore, Assertion (A) is correct. Also, Reason (R) is correct.
22	Ans (a) As we know, all congruent triangles are similar but similar triangle needs not to be congruent.

COORDINATE GEOMETRY

IMPORTANT POINTS:

Important Points:

- The distance of a point from the y-axis is called its X-coordinate or Abscissa. The distance of a point from the X-axis is called its Y-coordinate or Ordinate.
- The coordinates of a point on the X-axis are of the form (x, 0)

  The coordinate of a point on the Y-axis are of the form (0, y).
- The distance between the points $(x_1, y_1), (x_2, y_2) = \sqrt{(\boldsymbol{x_2 - x_1})^2 + (\boldsymbol{y_2 - y_1})^2}$

  Note: If O is the origin, the distance of a point P(x, y) from the origin O(0, 0) is given by

  OP = $\sqrt{x^2 + y^2}$
- Section Formula:

  The coordinates of the points P(x, y) which divides the line segment joining A(x_1, y_1) and B(x_2, y_2) internally in the ratio $m_1 : m_2$ is given by:

  $$\left(\frac{\boldsymbol{m_1 x_2 + m_2 x_1}}{\boldsymbol{m_1 + m_2}}, \frac{\boldsymbol{m_1 y_2 + m_2 y_1}}{\boldsymbol{m_1 + m_2}}\right)$$
- Mid-point Formula:

  If P is the mid-point of line segment AB, then $m_1 = m_2$ and the coordinates of P are:

  $$\left(\frac{\boldsymbol{x_1 + x_2}}{\boldsymbol{2}}, \frac{\boldsymbol{y_1 + y_2}}{\boldsymbol{2}}\right)$$
- Coordinates of Centroid of the Triangle with vertices $(x_1, y_1), (x_2, y_2)\ and\ (x_3, y_3)$ is given by:

  $$\left(\frac{\boldsymbol{x_1 + x_2 + x_3}}{\boldsymbol{3}}, \quad \frac{\boldsymbol{y_1 + y_2 + y_3}}{\boldsymbol{3}}\right)$$

SECTION-A (MCQs)

MCQs (SOLVED)

1. The distance of the point P$(-6, 8)$ from the origin is-

(a). 8 units (b). $2\sqrt{7}$ units (c). 10 units (d). 6 units

Ans. Option (c) 10 units

Sol. Distance of point = $\sqrt{(-6-0)^2 + (8-0)^2} = \sqrt{100} = 10\ units$ (Using distance formula)

2. If the distance between the points (4, k) and (1, 0) is 5, then the possible values of k is/are?

(a) 4 (b) -4 (c) ± 4 (d) none of these

Ans. Option (c) ± 4

Sol. Distance between the points = $\sqrt{(4-1)^2 + (k-0)^2}$

$5 = \sqrt{9 + k^2}$

$25 = 9 + k^2$

$k^2 = 16 \Rightarrow k = \pm 4$

3. Three vertices of the parallelogram ABCD are A(1, 4), B(-2, 3) and C(5, 8). The ordinate of the fourth vertex D is-

(a) 8 (b) 9 (c) 7 (d) 6

Ans. Option (b) 9

Sol. Let coordinate of vertex D are (x, y).

We know that the diagonals of parallellogram bisect each other

So, Midpoint of diagonal AC = Midpoint of diagonal BD

$\left(\frac{1+5}{2}, \frac{4+8}{2}\right) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$

$(3, 6) = \left(\frac{-2+x}{2}, \frac{3+y}{2}\right)$

On comparing the ordinates both sides, $6 = \frac{3+y}{2}$

$12 = 3 + y$

$y = 9$

4. If (a, b) is the mid-point of the line segment joining the points A(10, -6) and B(k, 4) and $a - 2b = 18$, the value of k is-

(a) 30 (b) 22 (c) 4 (d) 40

Ans. Option (b) 22

Sol. Midpoint of AB = $\left(\frac{10+k}{2}, \frac{-6+4}{2}\right)$

$(a, b) = \left(\frac{10+k}{2}, \frac{-6+4}{2}\right)$

On comparing, a = $\frac{10+k}{2}$ and $b = \frac{-6+4}{2} = -1$

Putting in the given equation, $a - 2b = 18$

$\frac{10+k}{2} - 2(-1) = 18$

$\frac{10+k}{2} = 16 \Rightarrow 10 + k = 32 \Rightarrow k = 22$

5. ASSERTION & REASONING

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

Assertion (A): The value of y is 6, for which the distance between the points P(2, –3) and Q(10, y) is 10.
Reason (R): Distance between two given points A (x_1, y_1) and B (x_2, y_2) is given by $\sqrt{(\boldsymbol{x_2 - x_1})^2 + (\boldsymbol{y_2 - y_1})^2}$
Ans. Option (d) Assertion (A) is false but reason (R) is true.
Solution: Assertion is false

$$PQ = \sqrt{(2-10)^2 + (-3-y)^2}$$
$$10 = \sqrt{64 + 9 + y^2 + 6y}$$
$$100 = y^2 + 6y + 73$$
$$y^2 + 6y - 27 = 0$$
$$(y+9)(y-3) = 0$$

Values of y are -9 and 3

MCQs (UNSOLVED)

6. The distance between the points (0, 5) and (-5, 0) is-
(a) 5 units (b) $5\sqrt{2}$ units (c). $2\sqrt{5}$ units (d). 10 units

7. The line segemnt joining the points $P(-3, 2)$& $Q(5, 7)$ is divided by the x-axis in the ratio:
(a). 3:1 (b) 3:4 (c) 3:2 (d) 3:5

8. The point P on x-axis is equidistant from the points A(-1, 0) and B(5, 0) is:
(a) (2, 0) (b) (0, 2) (c) (3, 0) (d) (2, 2)

9. The ratio in which the line $3x + y - 9 = 0$ divides the line segment joining the points (1, 3) and (2, 7) is:
(a) 3:2 (b) 2:3 (c) 3:4 (d) 4:3

10. ASSERTION & REASONING

Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertion (A) is true but reason (R) is false.
(d) Assertion (A) is false but reason (R) is true.

ASSERTION: The coordinates off the points which divides the line segment joining A(2, -3) and B(-4, -6) into three equal parts are (0, -4) and (-2, -5)

REASON: The points which divide AB in the ratio 1:3 and 3:1 are called points of trisection of AB.

Section-B (2 marks each)

(SOLVED)

11. The point P on *x*-axis equidistant from the points A(– 1, 0) and B(5, 0) is?

Sol. Coordinates of A = (-1, 0)

Coordinates of B = (5, 0)

Let the coordinate of the required point be M(x, 0)

M is equidistant from A and B $\Rightarrow AM = BM$

$\sqrt{(-1-x)^2 + (0-0)^2} = \sqrt{(5-x)^2 + (0-0)^2}$

On squaring both sides, $(-1-x)^2 = (5-x)^2$

$$1 + x^2 + 2x = 25 + x^2 - 10x$$

$$1 + 2x = 25 - 10x$$

$$12x = 24 \Rightarrow x = 2$$

Required point is (2, 0)

Alternate solution: since both the points are lying on x-axis so equidistant point will be the midpoint of the line segment joining AB, which also lies of x-axis. Mid point of -1 and 5 is 2. So required point is (2, 0)

12. Find the ratio in which the segment joining the points (1, −3) and (4, 5) is divided by *x*-axis? Also find the coordinates of this point on *x*-axis.

Sol. Let the ratio be K:1 and the point of division be (x, 0)

Using Section formula between A and B with ratio K: 1

$$(x, 0) = \left(\frac{k \times 1 + 1 \times 4}{k+1}, \frac{k \times (-3) + 1 \times 5}{k+1}\right)$$

On comparing

$$x = \frac{k+4}{k+1}, \quad 0 = \frac{-3k+5}{k+1}$$

$$-3k + 5 = 0 \Rightarrow 3k = 5 \Rightarrow k = \frac{5}{3}$$

Putting value of k in x,

$$x = \frac{{}^{5}/_{3} + 4}{{}^{5}/_{3} + 1} \Rightarrow x = \frac{17}{8}$$

Required ratio = 5: 3 and point on x-axis be $(\frac{17}{8}, 0)$

13. If the point P(6, 2) divides the line segment joining A(6, 5) and B(4, *y*) in the ratio 3 : 1, then the value of *y* is?

Sol. Using Section formula between A and B with ratio 3 : 1

$$(6, 2) = \left(\frac{3 \times 6 + 1 \times 4}{3+1}, \frac{3 \times 5 + 1 \times y}{3+1}\right)$$

On comparing, $2 = \frac{8+y}{4} \Rightarrow 8 = 8 + y \Rightarrow y = 0$

(UNSOLVED)

14. The distance between the points ($a\cos\theta + b\sin\theta$, 0) and (0, $a\sin\theta - b\cos\theta$), is?

15. If the point P(k, 0) divides the line segment joining the points A(2, – 2) and B(– 7, 4) in the ratio 1 : 2, then the value of k is?

16. Write the coordinates of a point P on x-axis which is equidistant from the point A(−2, 0) and B(6, 0).

SECTION-C (3 MARKS EACH)

(SOLVED)

17. Find the coordinates of a point A, where AB is a diameter of the circle with centre (− 2, 2) and B is the point with coordinates (3, 4).

Sol. Let the coordinate of point A be (x , y)

Using mid-point formula, $(-2,2)=\left(\frac{x+3}{2},\frac{y+4}{2}\right)$

$$-2=\frac{x+3}{2},\ 2=\frac{y+4}{2}$$

$$x+3=-4,\quad y+4=4$$

$$x=-7,\quad y=0$$

Coordinate of A(-7, 0)

18. If A(4, 3), B(−1, y) and C(3, 4) are the vertices of a right triangle ABC, right angled at A, then find the value of y.

Sol. Triangle ABC is a right angled triangle right angled at A,

$$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$AB=\sqrt{(-1-4)^2+(y-3)^2}$$

$$AB=\sqrt{(-5)^2+(y-3)^2}$$

$$AB=\sqrt{25+y^2+9-64}$$

$$AB=\sqrt{34+y^2-6y}$$

$$BC=\sqrt{(3-(-1))^2+(4-y)^2}$$

$$BC=\sqrt{(4)^2+(4-y)^2}$$

$$BC=\sqrt{16+16+y^2-8y}$$

$$BC=\sqrt{32+y^2-8y}$$

$$AC=\sqrt{(3-4)^2+(4-3)^2}$$

$$AC=\sqrt{(-1)^2+(1)^2}$$

$$AC=\sqrt{1+1}$$

$$AC=\sqrt{2}\ \text{units}$$

Given, ΔABC is a right angled triangle

So, by Pythagoras theorem $BC^2=AB^2+AC^2$

$$(\sqrt{32+y^2-8y})^2 = (\sqrt{2})^2 + (\sqrt{34+y^2-6y})^2$$

$$32 + y^2 - 8y = 2 + 34 + y^2 - 6y$$

$$-2y = 4$$

$$y = -2$$

Hence the value of y = -2.

19. Find the ratio in which the line $x - 3y = 0$ divides the line segment joining the points (−2, − 5) and (6, 3). Find the coordinates of the point of intersection.

Sol. Let the ratio of division be K : 1 and the point of division as (x, y)

Using Section formula, $(x, y) = \left(\frac{k\times(-2)+1\times6}{k+1}, \frac{k\times(-5)+1\times3}{k+1}\right)$

$$(x, y) = \left(\frac{-2k+6}{k+1}, \frac{-5k+3}{k+1}\right)$$

On comparing $x = \frac{-2k+6}{k+1}$, $y = \frac{-5k+3}{k+1}$

Now, the point of division lies on the given line $x - 3y = 0$. So the point (x, y) must satisfy the equation of the line,

$\Rightarrow \left(\frac{-2k+6}{k+1}\right) - 3\left(\frac{-5k+3}{k+1}\right) = 0$

$\Rightarrow \frac{-2k+6+15k-9}{k+1} = 0$

$\Rightarrow 13k - 3 = 0 \Rightarrow k = \frac{3}{13}$

For Point of division, putting value of k in x and y

After soolving $x = \frac{9}{2}, y = \frac{3}{2}$

Required ratio = 3: 13 and point of intersection (or division) = $\left(\frac{9}{2}, \frac{3}{2}\right)$

20. A line intersects the *y*-axis and *x*-axis at the points *P* and *Q* respectively. If (2, −5) is the mid-point of *PQ*, then find the co-ordinates of *P* and *Q*.

Sol. Let coordinate of P = (0, y)

Coordinate of Q = (x, 0)

Midpoint of PQ = $\frac{0+x}{2}, \frac{y+0}{2}$

$(2, -5) = \left(\frac{x}{2}, \frac{y}{2}\right)$

On comparing $x = 4, y = -10$

Required coordinate of P = (0, -10) and Q = (4, 0)

(UNSOLVED)

21. Let *P* and *Q* be the points of trisection of the line segment joining the points *A*(2, −2) and *B*(−7, 4) such that *P* is nearer to *A*. Find the coordinates of *P* and *Q*.

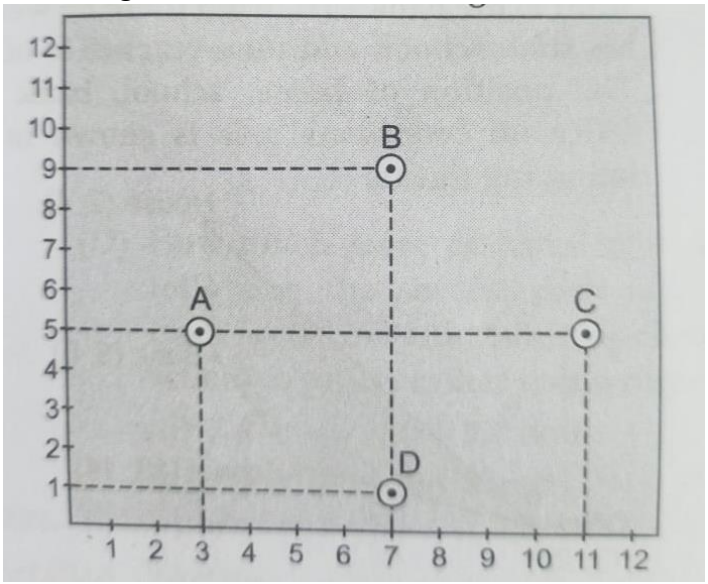
22. Find the ratio in which *y*-axis divides the line segment joining the points *A*(5, – 6) and *B*(–1, –4). Also find the coordinates of the point of division.

23. The *x*-coordinate of a point *P* is twice its *y*-coordinate. If *P* is equidistant from *Q* (2, −5) and *R*(−3, 6), find the coordinates of *P*.

24. If the points $A(1,-2), B(2,3), C(a,2)\ and\ D(-4,-3)$ form a parallelogram, then find the value of a and height of the parallelogram taking AB as Base.

CASE STUDY QUESTIONS (SOLVED)

25. Students of a school are standing in rows and columns in their school play ground for parade practice for Republic Day Celebration. A, B, C and D are the positions of four students as shown in the figure?



Now answer the following questions:

(i). What is the distance between the position of A and B?

(a) 4 units (b) $4\sqrt{2}$ units (c) 8 units (d) $3\sqrt{2}$ units

Ans. Option (b) $4\sqrt{2}$ units

Sol. Coordinate of A are (3, 5)

Coordinate of B are (7, 9)

∴ Distance AB = $\sqrt{(7-3)^2+\ (9-5)^2}$

= $\sqrt{16+16}$

` = $\sqrt{32}$ = $4\sqrt{2}$ units

(ii). Does the Quadrilateral ABCD forms a particular shape?

(a) Square (b) Rectangle (c) Rhombus (c) Trapezium

Ans. Option (a) Square

Sol. Distance AB = $4\sqrt{2}$ units

Distance BC = = $\sqrt{(11-7)^2+(5-9)^2} = \sqrt{32} = 4\sqrt{2}$units

Similarly, Distance CD and AD = $4\sqrt{2}$ units

Now, BD = $\sqrt{(7-7)^2+(9-1)^2} = \sqrt{64} = 8$ units

AC = $\sqrt{(11-3)^2+(5-5)^2} = \sqrt{64} = 8$units

Here, AB = BC = CD = AD and AC = BD

∴ ABCD is a square.

(iii). what would be the position of a point which is equidistant from all four points A, B, C and D, where a flag will be placed?

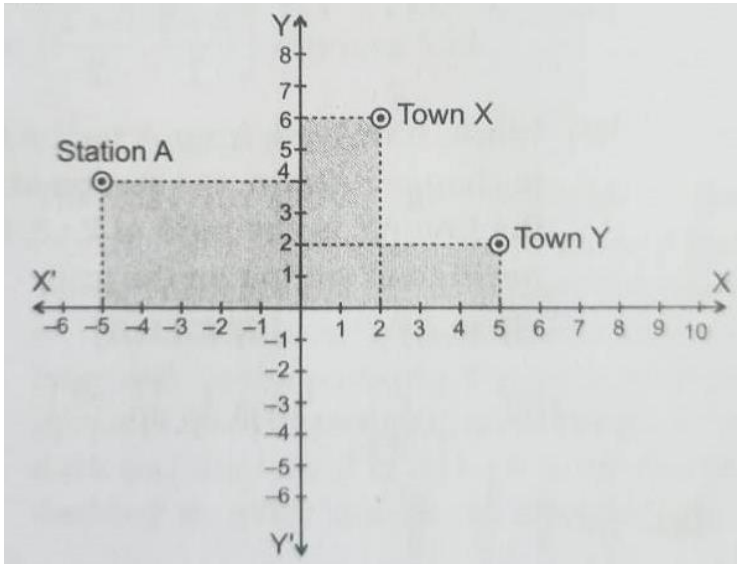
(a). (4, 4) (b) (3, 7) (c) (5, 8) (d) (7, 5)

Ans. Option (d) (7, 5)

Sol. As ABCD is a square, then mid-points of diagonals AC and BD will be a point equidistant from all the points.

∴ Mid point of AC = $\left(\frac{11+3}{2}, \frac{5+5}{2}\right)$

26. Two friends Danny an Alice works in the same call centre in Gurgaon. In the christmas vacation, the both decided to go to their hometowns represented by Town X and Town Y respectively. Town X and Town Y are connected by trains from the same station A in gurgaon. The situation of Town X, Town Y and station A is shown on the coordinate axis.



Now, answer the following questions:

(i). What is distance that Danny will travel to reach his hometown X?

(a). $\sqrt{51}$ units (b) $\sqrt{47}$ units (c) $\sqrt{53}$ units (d) $\sqrt{35}$ units

Ans. Option (c) $\sqrt{53}$ units

Sol. Coordinate of A = (-5, 4)

Coordinate of X = (2, 6)

∴ Distance = $\sqrt{(-5-2)^2+(4-6)^2}$

$= \sqrt{53}$

(ii). What is the distance that Alice will travel to reach her hometown?

(a) $2\sqrt{10}$units (b). $2\sqrt{26}$ units (c) $\sqrt{107}$units (d) $\sqrt{51}$units

Ans. Option (b) $2\sqrt{26}$ units

Sol. Coordinate of A = (-5, 4)

Coordinate of Y = (5, 2)

∴ Distance = $\sqrt{(-5-5)^2 + (4-2)^2}$

$= \sqrt{104} = 2\sqrt{26}$ units

(iii). Now, both of them plan to meet at a place between Town X and Town Y, such that it is a mod-point between both. Calculate the coordinates of the mid-point of X and Y.

(a) (1, 3) (b) (3.5, 4) (c) 2, 3) (d) (3, 3)

Ans. Option (b) (3.5, 4)

Sol. Coordinate of X = (2, 6)

Coordinate of Y = (5, 2)

Mid-point = $\left(\frac{2+5}{2}, \frac{6+2}{2}\right)$ = (3.5, 4)

(iv). While travelling from A to Y, Alice had to change its train, at a station, it divides the line AY in the ratio of 2: 3, find the position of station on the grid.

(a) (0, 3) (b) (-1.5, 2) (c) $\left(\frac{3}{8}, \frac{7}{9}\right)$ (d) $\left(-\frac{11}{5}, \frac{24}{5}\right)$

Ans. Option (d) $\left(-\frac{11}{5}, \frac{24}{5}\right)$

Sol. Coordinate of A = (-5, 4)

Coordinate of X = (2, 6)

∴ Coordinate of P = $\left(\frac{3\times(-5)+2\times2}{5}, \frac{4\times3+6\times2}{5}\right) = \left(-\frac{11}{5}, \frac{24}{5}\right)$

(v) The distance of the point P(x, y) from the origin is -……

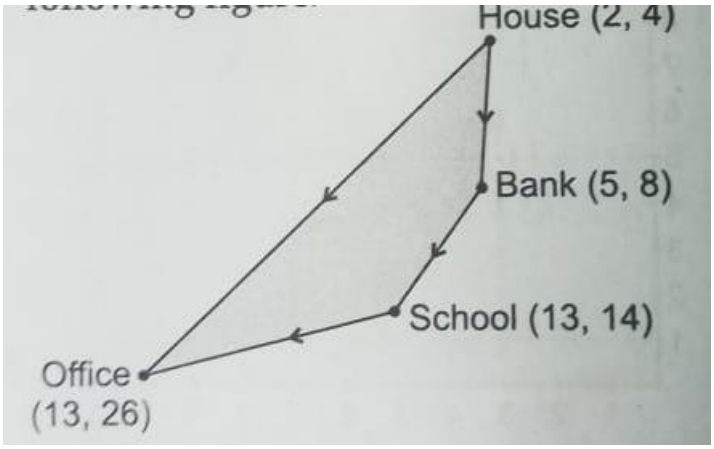
(a) $x^2 + y^2$ (b) $x^2 - y^2$ (c) $\sqrt{x^2 + y^2}$ (d) $\sqrt{x^2 - y^2}$

Ans. Option (c) $\sqrt{x^2 + y^2}$

Sol. Distance = $\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + y^2}$

CASE STUDY (UNSOLVED)

27. Rajesh went out from his house to reach the office. But he had to get some work done before going to the office. So he first of all went to the bank first, from there he went to his son's school, and then reaches to office. The position of home, school, bank and office on coordinate axis is shown in the following figure.



Now Answer the following questions:

(i). If Rajesh goes directly from bank to his office, how much distance he would travel?

(a) $2\sqrt{97}$ units (b) $4\sqrt{45}$units (c) $3\sqrt{91}$ units (d) $7\sqrt{7}$ units

(ii). If at the mid-point of the bank and school, there is an park, what are the coordinates if the park?

(a) (13, 11) (b) (-5, 10) (c) (9, 11) (d) (10, 12)

(iii). How much distance he will travel, if goes directly from home to the office?

(a) $15\sqrt{7}$ units (b) 15 units (c) $11\sqrt{5}$ units (d) $12\sqrt{3}$ units

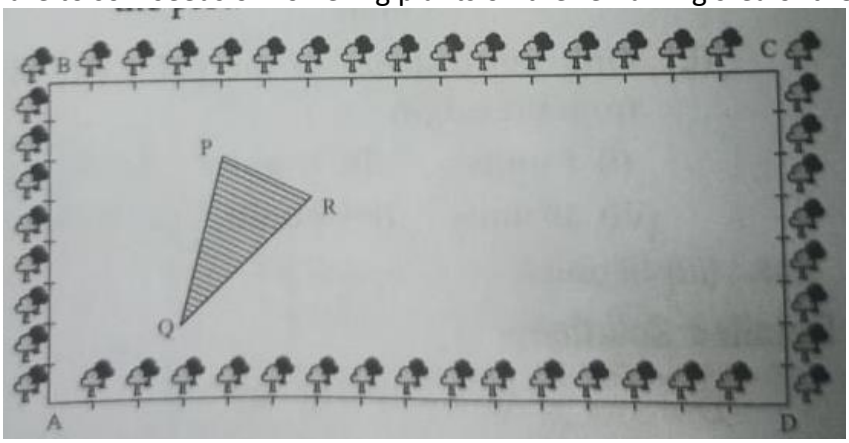
(iv). How much more distance he travel in following the second path i.e. going from house to bank, bank to school and school to office, rather going directly.

(a) 2.4 units (b) 5.7 units (c) 7.3 units (d) 8 units

(v) Find the distance of the point (-6, 8) from the origin

(a) 8 units (b) 11 units (c) 10 units (d) 9 units

28. The class X students in krishanagar have been sllotted a rectangular plot of land for their gardening activity. Saplings of Gulmohar are planted on the boundary at a distance of 1m from each other. There is triangular grassy lawn in the plot as shown in the figure. The students are to sow seeds of flowering plants on the remaining srea of the plot.



(i). Taking A as origin, the coordinates of P:

(a) (4, 6) (b) (6, 4) (c) 0, 6) (d) 4, 0)

(ii) What will be the coordinates of R, If C is the origin?

(a) (8, 6) (b) (3, 10) (c) (10, 3) (d) (0, 6)

(iii). What will be the coordinates of Q, if C is the origin?

(a) (6, 13) (b) (-6, 13) (c) (-13, 6) (d) (13, 6)

(iv). The length of side PR is- (considering A as Origin)

(a) $\sqrt{5}$ Units (b) $2\sqrt{5}$ units (c) $2\sqrt{2}$ units (d) none of these

INTRODUCTION TO TRIGONOMETRY

IMPORTANT POINTS:

1. In a right triangle ABC, right-angled at B,

$$\sin A = \frac{\text{side opposite to angle A}}{\text{hypotenuse}}, \cos A = \frac{\text{side adjacent to angle A}}{\text{hypotenuse}}$$

$$\tan A = \frac{\text{side opposite toangle A}}{\text{side adjacent to angle A}}$$

2. $\text{cosec } A = \frac{1}{\sin A}$; $\sec A = \frac{1}{\cos A}$; $\tan A = \frac{1}{\cot A}$, $\tan A = \frac{\sin A}{\cos A}$.
3. If one of the trigonometric ratios of an acute angle is known, the remaining trigonometric ratios of the angle can be easily determined.
4. The values of trigonometric ratios for angles 0°, 30°, 45°, 60° and 90°.

Ratios \ Angles	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
cosec θ	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
sec θ	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
cot θ	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

5. The value of sin A or cos A never exceeds 1, whereas the value of sec A or cosec A is always greater than or equal to 1.
6. $\sin^2 A + \cos^2 A = 1$,
   $\sec^2 A - \tan^2 A = 1$ for $0° \le A < 90°$,
   $\text{cosec}^2 A = 1 + \cot^2 A$ for $0° < A \le 90°$.

MCQ

Choose the correct answer from the given four options:

1. $\sec^2 45^0 - \tan^2 45^0 = ?$

a)0 b) 1 c) - 1 d) √2

Sol: b) $\sec^2 45^0 - \tan^2 45^0 = 1$ (according to identity $1+ \tan^2 \theta = \sec^2 \theta$)

2. If $\sin A + \sin^2 A = 1$, then find the value of $\cos^2 A + \cos^4 A$.

(a) 1 (b) 2 (c) 5 (d) 2

Sol : a) We have, $\sin A + \sin^2 A = 1$

$\sin A = 1 - \sin^2 A = \cos^2 A$ [as, $\sin^2\theta + \cos^2\theta = 1$)

$\sin A = \cos^2 A$

On squaring both sides, we get

$\sin^2 A = \cos^4 A$

$1 - \cos^2 A = \cos^4 A$

$\cos^2 A + \cos^4 A = 1$

3. $[\cos^4 A - \sin^4 A]$ is equal to:

(a) $2\sin^2 A - 1$ (b) $2\sin^2 A + 1$ (c) $2\cos^2 A - 1$ (d) $2\cos^2 A + 1$

Sol: c) $\cos^4 A - \sin^4 A$

$= \cos^4 A - (1 - \cos^2 A)^2$

$= \cos^4 A - (1 + \cos^4 A - 2\cos^2 A)$

$= 2\cos^2 A - 1$

4. If $\cos A = \frac{4}{5}$, then the value of tan A is

(a) $\frac{3}{5}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{4}{3}$ **(d)** $\frac{5}{3}$

5.Evaluate (cosec θ – cot θ) (cosec θ – cot θ).

a) 0

b) 1

c) 2

d) 3

6.If θ is said to be an acute angle and 7 $\sin^2$ θ + 3 $\cos^2$ θ = 4,then what is the value of tan θ?

a. 1

b. √3

c. 1/√3

d. None of the above

Answer: (c) 1/√3

solution: Given 7 $\sin^2$ θ + 3 $\cos^2$ θ = 4

=> 7 $\sin^2$ θ + 3 (1 - $\sin^2$ θ) = 4

=> 7 $\sin^2$ θ + 3 - 3$\sin^2$ θ = 4

Then, 4$\sin^2$ θ = 1

Or, sin θ = 1/2

So, θ = 30^0

Now, put θ = 30^0 in tan θ, we will get,

tan θ = 1/√3

7.If sin A + $\sin^2$A = 1, then the value of the expression ($\cos^2$A + $\cos^4$A) is

(a) 1

(b) 1/2

(c) 2

(d) 3

Answer: **(a) 1**

Explanation:

Given,

$\sin A + \sin^2 A = 1$

$\sin A = 1 - \sin^2 A$

$\sin A = \cos^2 A$ {since $\sin^2\theta + \cos^2\theta = 1$}

Squaring on both sides, $\sin^2 A = (\cos^2 A)^2$

$1 - \cos^2 A = \cos^4 A$

$\Rightarrow \cos^2 A + \cos^4 A = 1$

8.If sin θ + cos θ = 7/5, then sinθ cosθ is?

a. 11/25

b. 12/25

c. 13/25

d. 14/25

Answer : b. 12/25

solution:

Given: $\sin\theta + \cos\theta = 7/5$

By, squaring both sides of the above equation we get,

$\Rightarrow (\sin\theta + \cos\theta)^2 = 49/25$

$\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta = 49/25$

As we know that $\sin^2\theta + \cos^2\theta, = 1$

$\Rightarrow 1 + 2\sin\theta\cos\theta = 49/25$

$\Rightarrow 2\sin\theta\cos\theta = 24/25$

$\therefore \sin\theta \cos\theta = 12/25$

9.In Δ ABC, right-angled at B, AB = 24 cm, BC = 7 cm. The value of tan C is:

(a) 12/7 (b) 24/7 (c) 20/7 (d) 7/24

Sol: b

10.(Sin 30°+cos 60°)-(sin 60° + cos 30°) is equal to:

(a) 0 (b) 1+2√3 (c) 1-√3 (d) 1+√3

Answer: **(c) 1-√3**

11. The value of tan 60°/cot 30° is equal to:

(a) 0 (b) 1 (c) 2 (d) 3

Answer: **(b) 1**

12. $2\tan 30°/(1 + \tan^2 30°)$ =

(a) sin 60° (b) cos 60° (c) tan 60° (d) sin 30°

Answer: **(a) sin 60°**

Assertion and Reason Questions

1.Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).

(C) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

Assertion: The value of Sin A is always less than one.

Reasoning: SinA is the product of Sin and A

Answer: c

2.**Assertion :** The value of sec A (1 – sin A)(sec A + tan A) is 1.

Reason : $1 + \tan^2 A = \sec^2 A$, for any value of A.

(a) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion
(b) Both Assertion and Reason are correct and Reason is not the correct explanation for Assertion.
(c) assertion is true but the reason is false.
(d) both assertion and reason are false
Sol: a.) Both Assertion and Reason are correct and Reason is the correct explanation for Assertion.
For Reason, we know that $1 + \tan^2 A = \sec^2 A$,
for any value of A. So, Reason is correct

For Assertion, we have sec A (1 – sin A)(sec A + tan A)

= (sec A – secA.sinA)(sec A + tan A) = (sec A – tan A)(sec A + tan A) = $\sec^2 A - \tan^2 A = 1$

SHORT ANSWER TYPE QUESTIONS (2 marks)

1.Prove that $(1+\tan^2 A)(1-\sin A)(1+\sin A)=1$

Solution: $\sec^2 A (1 - \sin^2 A)$

$=\sec^2 A \cos^2 A$

$=\sec^2 A \times \frac{1}{\sec^2 A}$

=1

2 $\frac{SinA}{1-CosA}$ = cosecA+cotA

Solution

LHS=$\frac{SinA}{1-CosA} \times \frac{1+CosA}{1+CosA}$

= $\sin A(1+\cos A)/(1-\cos^2 A)$

$=\sin A(1+\cos A)/\sin^2 A$

=1/sinA+cosA/sInA =cosecA+ cotA PROVED

3. If $\sin\theta + \cos\theta = \sqrt{3}$, then find the value of $\sin\theta.\cos\theta$

Sol: $\sin\theta + \cos\theta = \sqrt{3}$

Squaring both side

$= \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$= 1 + 2\sin\theta\cos\theta = 3$

$= \sin\theta.\cos\theta = 1$

4. if $\sin\alpha = \frac{1}{\sqrt{2}}$ and $\cot\beta = \sqrt{3}$, then find the value of $\text{cosec}\alpha + coesc\beta$

Sol: $\text{cosec}\alpha = \frac{1}{\sin\alpha} = \sqrt{2}$

$= \text{cosec}\beta = \sqrt{1 + cot^2\beta}$

$= \sqrt{1+3} = 2$

$= \text{cosec}\alpha + coesc\beta = \sqrt{2} + 2$

5. Prove that $\sqrt{(1 - cos^2\theta)sec^2\theta} = \tan\theta$

Sol: $\sqrt{(1 - cos^2\theta)sec^2\theta}$

$= \sqrt{(sin^2\theta)sec^2\theta}$

$= = \sqrt{\frac{sin^2\theta}{cos^2\theta}}$

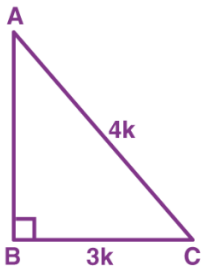
$= \sqrt{tan^2\theta}$

$= \tan\theta$

6.If Sin A = 3/4, Calculate cos A and tan A.

Solution: Sin A = 3/4

Sin A = Opposite Side/Hypotenuse Side = 3/4



Let BC be 3k and AC will be 4k.

Acc to Pythagoras theorem

Hypotenuse^2 = Perpendicular^2+ Base^2

$AC^2 = AB^2 + BC^2$

$(4k)^2 = (AB)^2 + (3k)^2$

$16k^2 – 9k^2 = AB^2$

$AB^2 = 7k^2$

Hence, AB = √7 k

cos A = Adjacent Side/Hypotenuse side = AB/AC

cos A = √7 k/4k = √7/4

And,

tan A = Opposite side/Adjacent side = BC/AB

tan A = 3k/√7 k = 3/√7

7.If $x = a \cos \theta – b \sin \theta$ and $y = a \sin \theta + b \cos \theta$, then prove that $a^2 + b^2 = x^2 + y^2$.

Solution:

R.H.S. = $x^2 + y^2$

$= (a \cos \theta – b \sin \theta)^2 + (a \sin \theta + b \cos \theta)^2$

$= a^2\cos^2 \theta + b^2 \sin^2 \theta – 2ab \cos \theta \sin \theta + a^2 \sin^2 \theta + b^2 \cos^2 \theta + 2ab \sin \theta \cos \theta$

$= a^2(\cos^2 \theta + \sin^2\theta) + b^2 (\sin^2 \theta + \cos^2 \theta)$

$= a^2 + b^2$ = L.H.S. …[$\because \cos^2 \theta + \sin^2 \theta = 1$

8.If tan α = 3–√ and tan β = 13√,0 < α, β < 90°, find the value of cot (α + β).

Solution:

tan α = 3–√ = tan 60° …(i)

tan β = 13√ = tan 30° …(ii)

Solving (i) & (ii), α = 60° and β = 30°

∴ cot (α + β) = cot (60° + 30°) = cot 90° = 0

Short answer types (3 marks)

1. tan θ sin θ + cos θ = sec θ

Solution:= tan θ sin θ + cos θ

$=\frac{\sin\theta}{\cos\theta}$ x sin θ + cos θ

$=\frac{(\sin\theta)^2}{\cos\theta}$ + cos θ

=sin ^{2}θ +cos^2 θ/cos θ (sin ^{2}θ +cos^2 θ=1)

=1/cos θ

=secθ

2. $\frac{1+sinA}{1-sinA}$ = (secA+tanA)^2

Solution

$\frac{1+sinA}{1-sinA}$x$\frac{1+sinA}{1+sinA}$

=(1+sinA)^2/ 1-sin^{2}A

$=(1+\sin A)^2 \ / \cos^2 A$ (by $1-\sin^2 A= \cos^2 A$)

$=(\sec A +\tan A)^2$ Ans

3.If $7\sin^2 A + 3\cos^2 A = 4$, show that $\tan A = 1/\sqrt{3}$

Solution: on dividing both sides by $\cos^2 A$

$$:\frac{7\sin^2 A + 3\cos^2 A}{\cos^2 A}=\frac{4}{\cos^2 A}$$

$7\tan^2 A + 3 = 4\sec^2 A$

$7\tan^2 A + 3 = 4(\tan^2 A +1)$

4. Prove that $\frac{\boldsymbol{tan\theta+\sec\theta-1}}{\boldsymbol{tan\theta-\sec\theta+1}} = \frac{\boldsymbol{1+sin\theta}}{\boldsymbol{cos\theta}}$

Sol: LHS $=\frac{tan\theta+\sec\theta-1}{tan\theta-\sec\theta+1}$

We can write, $\sec^2\theta-\tan^2\theta = 1$

$$= \frac{(\tan\theta+\sec\theta)-(\sec^2\theta-\tan^2\theta)}{\tan\theta-\sec\theta+1}$$

$$=\frac{(\tan\theta+\sec\theta)-(1-\sec\theta+\tan\theta)}{\tan\theta-\sec\theta+1}$$

$= \tan\theta + \sec\theta$

$= \frac{1+\sin\theta}{\cos\theta}$ = RHS

5. prove that : $\frac{\boldsymbol{\tan\theta}}{\boldsymbol{1-\cot\theta}}+\frac{\boldsymbol{\cot\theta}}{\boldsymbol{1-\tan\theta}}$ = $\boldsymbol{1+\sec\theta\,cosec\theta}$

Sol: $\frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} + \frac{\cos^2\theta}{\sin\theta(\cos\theta-\sin\theta)}$

$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta(\sin\theta-\cos\theta)}$

$= \frac{\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta}{\sin\theta\cos\theta}$

$= \frac{1}{\sin\theta\cos\theta} + 1$

$= 1+\text{cosec }\theta\sec\theta$

6. Prove that $\frac{\mathbf{1+secA}}{\mathbf{secA}} = \frac{\mathbf{sin^2A}}{\mathbf{1-cosA}}$

Sol: LHS = $\frac{1+\text{secA}}{\text{secA}}$ = $\frac{1+\frac{1}{cosA}}{\frac{1}{cosA}}$

=1+cosA

= $\frac{(1-cosA)(1+cosA)}{(1-cosA)}$

=$\frac{1-cos^2A}{1-cosA}$

= $\frac{sin^2A}{1-cosA}$ = RHS

7.If tan (A + B) =√3 and tan (A – B) =1/√3, 0° < A + B ≤ 90°; A > B, find A and B.

Solution: Given, tan (A + B) = √3

tan 60° = √3

⇒ tan (A + B) = tan 60°

⇒(A + B) = 60° …… (i)

⇒ tan (A – B) = tan 30°

⇒(A – B) = 30° ….. (ii)

Adding the equation (i) and (ii), we get;

A + B + A – B = 60° + 30°

2A = 90°

A= 45°

Now, put the value of A in eq. (i) to find the value of B;

45° + B = 60°

B = 60° – 45°

B = 15°

Therefore A = 45° and B = 15°

8.If $x = p \sec \theta + q \tan \theta$ and $y = p \tan \theta + q \sec \theta$, then prove that $x^2 - y^2 = p^2 - q^2$.

Solution:

L.H.S. = $x^2 - y^2$

= $(p \sec \theta + q \tan \theta)^2 - (p \tan \theta + q \sec \theta)^2$

= $p^2 \sec \theta + q^2 \tan^2 \theta + 2 pq \sec^2 \tan^2 -(p^2 \tan^2 \theta + q^2 \sec^2 \theta + 2pq \sec \theta \tan \theta)$

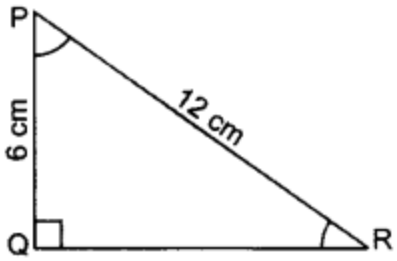
= $p^2 \sec \theta + 2 \tan^2 \theta + 2pq \sec \theta \tan \theta - p^2 \tan^2 \theta - q^2 \sec \theta - 2pq \sec \theta \tan \theta$

= $p^2(\sec^2 \theta - \tan^2 \theta) - q^2(\sec?^2 \theta - \tan^2 \theta) =$

= $p^2 - q^2$ …[$\sec^2 \theta - \tan^2 \theta = 1$

= R.H.S.

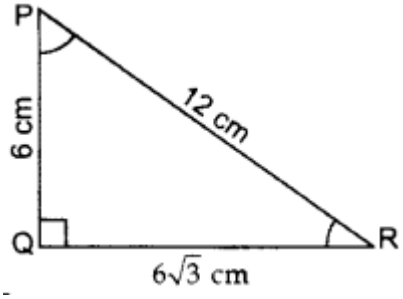
9.In figure, ΔPQR right angled at Q, PQ = 6 cm and PR = 12 cm. Determine ∠QPR and ∠PRQ.



Solution:

In rt. ΔPQR,

$PQ^2 + QR^2 = PR^2$ …[By Pythogoras' theorem



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$(6)^2 + QR^2 = (12)^2$

$QR^2 = 144 - 36$

$QR^2 = 108$

$QR = \sqrt{36 \times 3} = 6\sqrt{3}$ cm

$\tan R = \frac{PQ}{QR}$	$\tan P = \frac{QR}{PQ}$
$\tan R = \frac{\cancel{6}}{\cancel{6}\sqrt{3}} = \frac{1}{\sqrt{3}}$	$\tan P = \frac{\cancel{6}\sqrt{3}}{\cancel{6}} = \sqrt{3}$
$\tan R = \tan 30^\circ$	$\tan P = \tan 60^\circ$
$R = 30^\circ$	$P = 60^\circ$
$\angle PRQ = 30^\circ$	$\angle QPR = 60^\circ$

LONG ANSWER TYPES (5 MARKS)

1. $(1 + \tan^2 A/1 + \cot^2 A) = (1 - \tan A/1 - \cot A)^2 = \tan^2 A$

solution

LHS:

$= (1+\tan^2 A) / (1+\cot^2 A)$

Using the trigonometric identities we know that $1+\tan^2 A = \sec^2 A$ and $1+\cot^2 A = \operatorname{cosec}^2 A$

$= \sec^2 A / \operatorname{cosec}^2 A$

On taking the reciprocals we get

$= \sin^2 A/\cos^2 A$

$= \tan^2 A$

RHS:

$=(1-\tan A)^2/(1-\cot A)^2$

Substituting the reciprocal value of tan A and cot A we get

$= (1-\sin A/\cos A)^2/(1-\cos A/\sin A)$

$= [(\cos A-\sin A)/\cos A]^2/ [(\sin A-\cos)/\sin A)^2] = [(\cos A-\sin A)^2 \times \sin^2 A] /[\cos^2 A. /(\sin A-\cos A)^2]$

$= \sin^2 A/\cos^2 A$

$= \tan^2 A$

The values of LHS and RHS are the same.

Hence proved.

2. **Prove that (sin A – 2 $\sin^3$A)/(2 $\cos^3$A – cos A) = tan A.**

Solution:

LHS = (sin A – 2 $\sin^3$A)/(2 $\cos^3$A – cos A)

= [sin A(1 – 2 $\sin^2$A)]/ [cos A(2 $\cos^2$A – 1]

Using the identity $\sin^2\theta + \cos^2\theta = 1$,

= [sin A($\sin^2$A + $\cos^2$A – 2 $\sin^2$A)]/ [cos A(2 $\cos^2$A – $\sin^2$A – $\cos^2$A]

= [sin A($\cos^2$A – $\sin^2$A)]/ [cos A($\cos^2$A – $\sin^2$A)]

= sin A/cos A

= tan A

= RHS

Hence proved.

3. if $x\sin^3\theta + y\sin^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\sin\theta$, prove that $x^2+y^2=1$

Sol: Given, x $\sin^3\theta$ + y $\cos^3\theta$ = sin θ cos θ

$=$ x sin θ ($\sin^2\theta$) + y cos θ ($\cos^2\theta$) = sin θ cos θ

$=$ x sin θ ($\sin^2\theta$) + x sin θ ($\cos^2\theta$) = sin θ cos θ

$=$ x sin θ ($\sin^2\theta + \cos^2\theta$) = sin θ cos θ

$=$ x = cos θ

Given, x sin θ = y cos θ

$=$ cos θ sin θ = y cos θ

$=$ y = sin θ

LHS = $x^2 + y^2 = (\cos\theta)^2 + (\sin\theta)^2 = 1$ = RHS

4. prove that $\frac{1+sec\theta-\tan\theta}{1+sec\theta+\tan\theta} = \frac{1-sin\theta}{cos\theta}$

Sol: taking LHS,

ON multiplying by (sec θ-tan θ) on numerator and denominator,

$\frac{1+sec\theta-\tan\theta}{1+sec\theta+\tan\theta}$

$$=\frac{(\mathbf{1}+\boldsymbol{sec}\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\mathbf{1}+\boldsymbol{sec}\theta+\tan\theta)(\sec\theta-\tan\theta)}$$

$$=\frac{(\mathbf{1}+\boldsymbol{sec}\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\boldsymbol{sec}\theta-\tan\theta)+(\sec\theta+\tan\theta)(\sec\theta-\tan\theta)}$$

$$=\frac{(\mathbf{1}+\boldsymbol{sec}\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\boldsymbol{sec}\theta-\tan\theta)(\boldsymbol{sec^2}\theta-\boldsymbol{tan^2}\theta)}$$

$$=\frac{(\mathbf{1}+\boldsymbol{sec}\theta-\tan\theta)(\sec\theta-\tan\theta)}{(\boldsymbol{sec}\theta-\tan\theta+1)}$$

$= sec\theta - \tan\theta$

$$=\frac{\mathbf{1}}{\boldsymbol{cos}\theta} - \frac{\boldsymbol{sin}\theta}{\boldsymbol{cos}\theta}$$

$$= \frac{1-sin\theta}{cos\theta}$$

5.(1 + tan²A/1 + cot²A) = (1 – tan A/1 – cot A)² = tan²A

Solution: Given: (1 + $\tan^2$A/1 + $\cot^2$A) = (1 – tan A/1 – cot A)^2 = $\tan^2$A

LHS:

= (1+$\tan^2$A) / (1+$\cot^2$A)

(1 + $\tan^2$A/1 + $\cot^2$A) = (1 – tan A/1 – cot A)^2 = $\tan^2$A

Solution: Given: (1 + $\tan^2$A/1 + $\cot^2$A) = (1 – tan A/1 – cot A)^2 = $\tan^2$A

LHS:

=(1+tan²A)/(1+cot²A)

Using the trigonometric identities, 1+tan²A = sec²A, 1+cot²A= cosec²A

= sec²A/ cosec²A

On taking the reciprocals we get

= sin²A/cos²A

= tan²A

RHS:

=(1-tanA)²/(1-cotA)²

Substituting the reciprocal value of tan A and cot A we get,

= (1-sinA/cosA)²/(1-cosA/sinA)²

= [(cosA-sinA)/cosA]²/ [(sinA-cos)/sinA)²] = [(cosA-sinA)²×sin²A] /[cos²A. /(sinA-cosA)²]

= sin²A/cos²A

= tan²A

The values of LHS and RHS are the same.

Hence proved.

APPLICATIONS OF TRIGONOMETRY

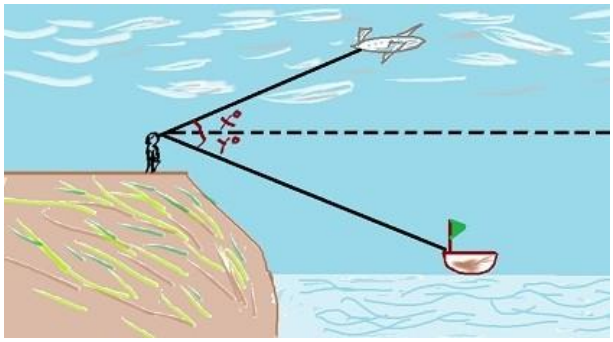
IMPORTANT POINTS:

Let us see some real-life applications of Trigonometry. Trigonometry is interesting and has many useful applications in the field of astronomy, geography etc. One of the useful applications of trigonometry is height and distance problem. Height and distance problems involve calculating various distances, angles, and heights in geometric scenarios, typically involving triangles. These problems often deal with objects like towers, buildings, ladders, and flags, and they require understanding the relationships between different elements such as angles of elevation and depression, distances, and heights.

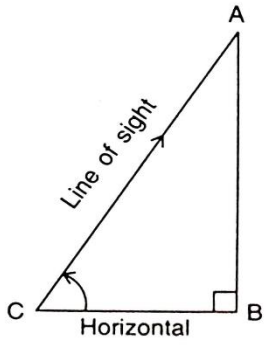
Definitions:

Let us define a few terminologies so that we can understand and implementthe concept of Trigonometry to find the heights and distances.

Line of sight is a straight line from our eye to the object.
If an object is below the horizontal line from the eye, we have to lowerour head to view the object. In this process our eyes move through an angle. This angle is called the **angle of depression**. The angle of depression of an object viewed is the angle formed by the line of sightwith the horizontal line, when the object is below the horizontal line.
Similarly, if an object is above the horizontal line from our eyes, we have to raise our head to view the object. In this process our eyes move through an angle formed by the line of sight and horizontal line which is called the **angle of elevation**.



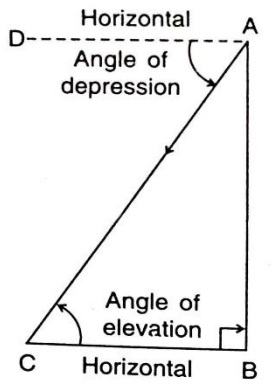
Here x^o is the angle of elevation and y^o is the angle of depression



Let AB be a tower (or pillar or minar etc.) standing on a level ground and a man, standing at any point C on the level ground, is viewing an object at A. The line CA, joining his eye to the object, is called the line of sight.
The angle, which the line of sight makes with the horizontal is called the angle of elevation. In the given figure; angle ACB is the angle of elevation.

Similarly, if the man is at A and is viewing an object C on the level ground, then the angle, which the line of sight (AC) makes with horizontal, is called the angle of depression.

In the given figure; angle DAC is angle of depression.
Angle of elevation of point A as seen from point C is equal to the angle of depression of point C as seen from point A. i.e., ∠ACB = ∠DAC.



Solved Problems

MCQs (1 mark)

1.

A tower casts a shadow 30 meters long. At the same time, a 2-meter-tall object nearby casts a shadow 3 meters long. What is the height of the tower?

A) 15 meters
B) 20 meters
C) 25 meters
D) 30 meters

Answer: B) 20 meters

Explanation: By similar triangles, height of tower = (Height of object/Shadow length of object) x Shadow length of tower = (2/3) x 30 = 20 meters.

2.

A ladder leans against a wall. The ladder is 10 meters long and makes a 60° angle with the ground. How far is the ladder from the base of the wall?

A) 5 meters
B) 7.5 meters
C) 10 meters
D) 15 meters

Answer: A) 5 meters

Explanation: Using trigonometry, distance from wall = Ladder length x cos(angle) = 10 x cos(60°) = 10 x 0.5 = 5 meters.

3.

From a point P on the ground, the angle of elevation to the top of a vertical pole is 45°. If the height of the pole is 10 meters, what is the distance of point P from the base of the pole?

A) 5√2 meters
B) 10√2 meters
C) 15 meters
D) 20 meters

Answer: A) 5√2 meters

Explanation: Using trigonometry, distance from base = Height of pole / tan(angle) = 10 / tan(45°) = 10 / 1 = 10 meters. Since the angle of elevation is 45°, the distance will be the same, i.e., 10 meters.

4.

An observer 1.6 meters tall is standing 20 meters away from a tree. The angle of elevation from his eyes to the top of the tree is 30°. What is the height of the tree?

A) 5 meters
B) 7 meters
C) 10 meters
D) 20 meters

Answer: C) 10 meters

Explanation: Using trigonometry, height of tree = (Observer's height + Distance from tree x tan(angle)) = (1.6 + 20 x tan(30°)) = (1.6 + 20 x √3/3) = (1.6 + 20 x 1.73/3) = (1.6 + 11.53) ≈ 13.13 meters.

5.The length of a string between a kite and a point on the ground is 85 m. If the string makes an angle q with the grou level such that tan q = 15/ 8 , then the kite is at what height from the ground ?

A) 50 meters
B) 75 meters
C) 10 meters
D) 20 meters

Ans- B) 75 meters

6. If the altitude of the Sun is at 60°, then find the height of the vertical tower that will cast a shadow of length 20 m

A) 20 meters
B) 30√3 meters
C) 25√3 meters
D) 20√3 meters

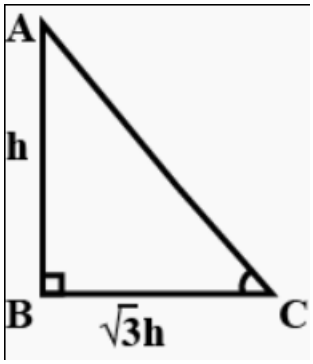
Ans : 20√3 m

7.The length of shadow of a tower on the plane ground is $\sqrt{3}$ is height of the tower. The angle of elevation of sun is: (a) 45° (b)30° (c) 60° (d) 90°

Correct option is B)

Let the height of the tower be h meter.

Then the length of the tower be 3h meter



∴ In △ABC

⇒tanθ=BC/AB

⇒tanθ=$\frac{h}{\sqrt{3}h}$

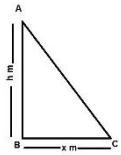
⇒tanθ=$\frac{1}{\sqrt{3}}$

⇒tanθ=tan30∘ [∵tan30∘=$\frac{1}{\sqrt{3}}$]

⇒θ=30∘.

8.The ratio of the length of tv tower and its shadow is √3: 1.The angle of elevation of the sun is
(a) 30° (b)45° (c) 60° (d) 90°

Correct option is (c)



Let the height of tower.AB=h m

The length of shadow BC=x m

According to given question,

$$\frac{AB}{BC} = \frac{\sqrt{3}}{1}$$
$$\frac{h}{x} = \frac{\sqrt{3}}{1}$$

We know that,

In ΔABC,

tanθ=$\frac{AB}{AC}$

tanθ=$\frac{h}{x}$

tanθ=$\frac{\sqrt{3}}{1}$

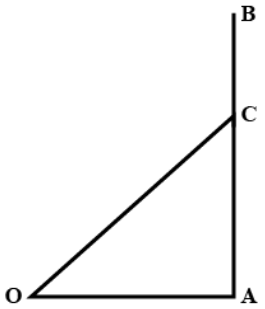
tanθ=√3

tanθ=tan60

θ=60°

9.A pole is broken by the storm of wind and its top struck the ground at an angle of 45° and at a distance of 25 m from the foot of the pole. The height of the pole before it was broken was ?
a) 25√2 m (b) 25(1+√2) m (c) 20√3 m (d) (25√3)/3 m

Correct option is B)



Let AB be the tree broken at C, such that the broken part CB takes the position CO and strikes the ground at O.

It is given that OA=25 m and ∠AOC=45°

Let AC=x and CB=y then CO= y

In △OAC, we have,

tan45° =AO/AC ⇒1=25/x ⇒x=25

Again in △OAC, we have

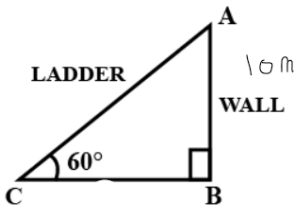
cos45°=$\frac{OC}{OA}$ ⇒√2=y/25 ⇒y=25√2

Height of the tree =(x+y)

=25+25√2=25(1+√2)

10.A ladder is lying/resting on a 10 m high wall. If it makes an angle of 60° with horizontal then the distance between foot of ladder & wall is

(a) 10/√3m (b) (20√3)/3 m (c) 10√3 m (d) 20√3 m



Correct option is C)

Let AC be the ladder and AB be the wall.

Then, BC is the distance between base of the ladder & wall. AB=10m

Also, ∠C = 60°

Now, In △ABC, tan 60°= AB/BC

∴ BC=10/√3

11.A minar is 800 m high from sea's surface. A guard sees a yacht of enemy from minar, which makes an angle of depression 60°. Find the distance between yacht and foot of the minar ?

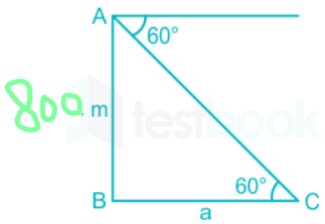
(a) 600 m (b) 180 √3 m (c) 800/√3 m (d) 160√3 m

Correct option is (c) 800/√3 m

tan60°=$\frac{AB}{BC}$

Or,√3=$\frac{800}{a}$

Or,a= $\frac{800}{\sqrt{3}}$m



12.The thread of a kite is 120 m long and it is making 30° angular elevation with the ground. What is the height of the kite?

(a) 60 m (b) 180 m (c) 80m (d) 160m

Ans-Correct option is (a)

Height of the kite = h meter

Length of the thread = 120 m

Given 30° angular elevation with the ground

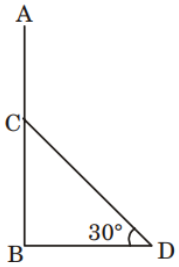
Sin (30) = h/120

1/2=h/120

Height of the kite (h) = 60 meter

13.A vertical post 35 ft high is broken at a certain height and its upper part, not completely separated, meets the ground at an angle of 30°. Find the height at which the post is broken.

(a) 50 ft (b) 70/6 ft (c) $15\sqrt{3}/(2-\sqrt{3})$ ft (d) $5\sqrt{3}$ ft



Ans-Correct option is (b)

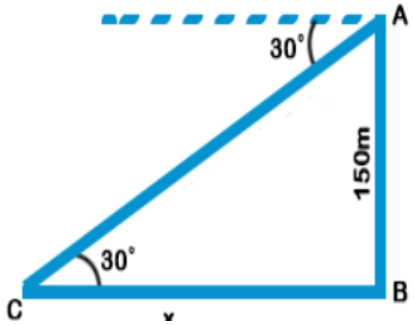
14.The angle of depression of car parked on the road from the top of 150 m high tower is 30°.The distance of the car from the tower (in metres)is

(a) $50\sqrt{3}$m (b) $150\sqrt{3}$m (c) $150\sqrt{2}$m (d)75m

Ans-Correct option is (b)$150\sqrt{3}$m

In right - angled triangle ABC,

$\frac{BC}{AB}=\cot 30^\circ \Rightarrow \frac{BC}{150}=\sqrt{3} \Rightarrow BC=150\sqrt{3}$ m



Assertion and Reason Questions

1.Assertion: If two objects are of different heights but cast shadows of equal lengths, they must be at the same distance from the light source.

Reasoning: The length of the shadow depends solely on the angle of elevation of the light source.

A) Both the assertion and reasoning are true and the reasoning is the correct explanation of the assertion.

B) Both the assertion and reasoning are true but the reasoning is not the correct explanation of the assertion.

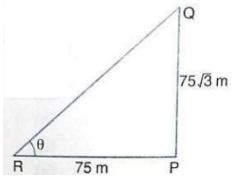
C) The assertion is true, but the reasoning is false.

D) Both the assertion and reasoning are false.

Answer: B) Both the assertion and reasoning are true but the reasoning is not the correct explanation of the assertion.

Explanation: While it is true that the length of the shadow depends on the angle of elevation of the light source, the assertion oversimplifies the situation. Even if two objects are at different distances from the light source, they can still cast shadows of equal length if their heights and the angles at which they are casting shadows are different.

2.Assertion: In the figure ,if QP=$75\sqrt{3}$ m ,PR=75 m ,then $\angle QRP =$ 60°



Reason: tanΘ= $\frac{QP}{PR}=\frac{perpendicular}{base}$, $\angle QRP = \Theta$, $\tan 60^{\circ}=\sqrt{3}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertions (A) is true but reason (R) is false.
(d) Assertions (A) is false but reason (R) is true.

Ans-(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

3.Assertion: The shadow of a tower is 15 m when the sun's elevation is 30° if the sun's elevation is 60° then the shadow of the tower 5m

Reason: tanΘ= $\frac{perpendicular}{base}$,$\tan 60^{\circ}=\frac{1}{\sqrt{3}}$, $\tan 30^{\circ}=\sqrt{3}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertions (A) is true but reason (R) is false.
(d) Assertions (A) is false but reason (R) is true.

Ans-(c) Assertions (A) is true but reason (R) is false.

4.Assertion: The shadow of a tower is 15 m when the sun's elevation is 30° if the sun's elevation is 60° then the shadow of the tower 5m

Reason: tanΘ= $\frac{perpendicular}{base}$,$\tan 60^{\circ}=\frac{1}{\sqrt{3}}$, $\tan 30^{\circ}=\sqrt{3}$

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertions (A) is true but reason (R) is false.
(d) Assertions (A) is false but reason (R) is true.
Ans-(c) Assertions (A) is true but reason (R) is false.

5.Assertion: If the length of shadow of a vertical pole and its height are in the ratio $\sqrt{3}$:1, then the angle of elevation of the sun is 60°

Reason: cotΘ= $\frac{base}{perpendicular}$,$\cot 30^{\circ}=\frac{\sqrt{3}}{1}$,

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertions (A) is true but reason (R) is false.
(d) Assertions (A) is false but reason (R) is true.

Ans-d) Assertions (A) is false but reason (R) is true.

6.Assertion: If the length of shadow of a vertical pole is equal to its height, then the angle of elevation of the sun is 45°
Reason: According to Pythagoras theorem, $h^2=p^2+b^2$, where h = hypotenuse, p=perpendicular and b = base

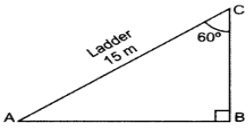
(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertions (A) is true but reason (R) is false.
(d) Assertions (A) is false but reason (R) is true
Ans: b) both assertion and reason are correct and reason is correct explanation for assertion.
7.Assertion: If shadow of pole is $\frac{1}{\sqrt{3}}$ of its height, then the altitude of the sun is 60°
Reason: If the sun's altitude is 45°,then the shadow of a vertical pole is same as height.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).
(b) Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A).
(c) Assertions (A) is true but reason (R) is false.
(d) Assertions (A) is false but reason (R) is true.
Ans: b) both assertion and reason are correct and reason is correct explanation for assertion.

Short Answer Type (2 marks)

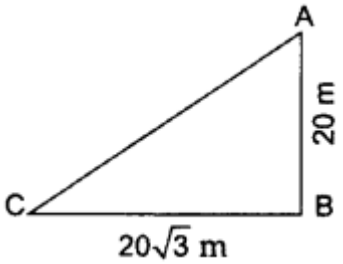
1. A ladder 15 m long just reaches the top of a vertical pole. If the ladder makes an angle of 60° with the pole, then calculate the height of the pole.

Solution:



∠BAC = 180° – 90° – 60° = 30°
sin 30° = BC/AC
1/2=BC/15
2BC = 15
BC = 15/2m = 7.5m

2. A tower AB is 20 m high and BC, its shadow on the ground, is 20√3 m long. Find the Sun's altitude.
Solution:
AB = 20 m, BC = 20√3 m,



∠ACB = θ ? In ΔABC,

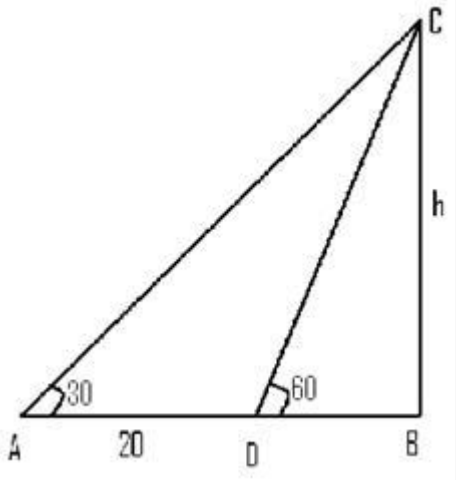
$$\frac{AB}{BC} = \tan\theta$$

$$\frac{20}{20\sqrt{3}} = \tan\theta$$

$$\frac{1}{\sqrt{3}} = \tan\theta$$

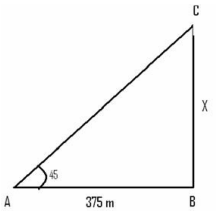
$$\tan\theta = \tan 30^\circ \quad \Rightarrow \quad \theta = 30^\circ$$

3. The angle of elevation of the top of a tower from point A on the ground is 30^0. On moving a distance of 20m towards the foot of the tower, the angle of elevation increases to 60^0. Find the height of the tower.



Let h be the height of tower
From the figure. 20 =h (cot30 - cot60) , 20 =h (√3-1/√3) => 20√3 = h (3-1)
=> h=10√3. Let h be the height of tower

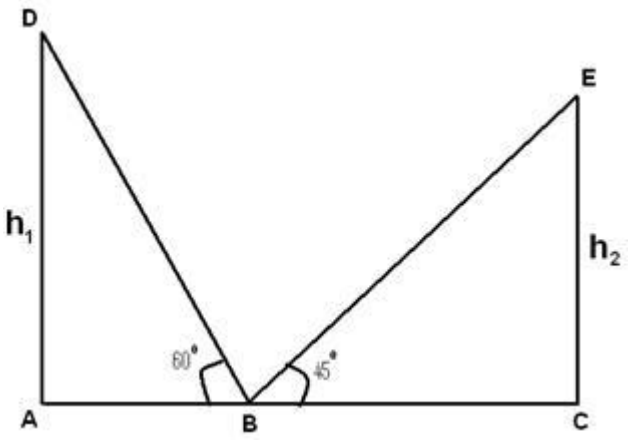
4. From a point 375 metres away from the foot of a tower, the top of the tower is observed at an angle of elevation of 45°, then find the height of
height of the. tower.



From the right angled triangle
Tan(45°)= X/375
=> X = 375 m

5. The angles of elevation of the tops of two vertical towers as seen from the middle point of the lines joining the foot of the towers are 45° & 60 ° . Find ratio of the height of the towers.

solution:



Tan(60)=h_1/AB

=> h1=√3AB

Tan(45)=h_1/BC

=> h2=BC

h1/ h2=√3/1

=> h1:h2=√3:1

PRACTICE QUESTIONS (2 marks)

1. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.
2. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.
3. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x : y.
4. From the top of a vertical tower, the angles of depression of two cars, in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. [Use √3= 1.732]
5. As observed from the top of a 60 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light-house, find the distance between the two ships. [Use √3= 1.732]

LONG ANSWER TYPE (05 Marks)

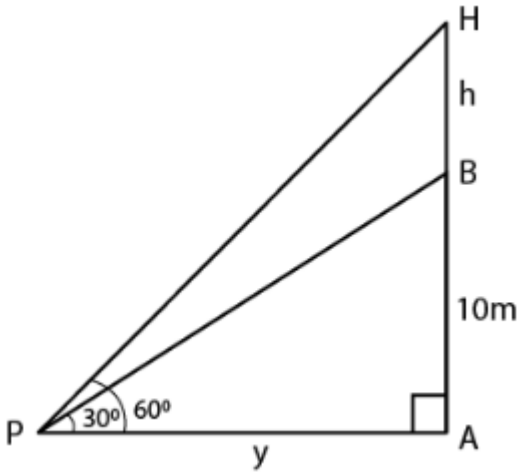
1. From a point P on the ground, the angle of elevation of the top of a 10 m tall building and a helicopter hovering over the top of the building are 30^0 and 60^0 respectively. Find the height of the helicopter above the ground.

Solution:

Consider AB as the building and H as the helicopter hovering over it

P is a point on the ground

The angle of elevation of the top of the building and helicopter are 30^0 and 60^0



We know that, Height of the building AB = 10 m

Take PA = x m and BH = h m

In right Δ ABP, tan θ = P/B, Substituting the values

Tan 30^0 = AB/PA = 10/x, so we get $1/\sqrt{3} = 10/x$

$x = 10\sqrt{3}$ m, in right Δ APH

Tan 60^0 = AH/PA, tan 60^0 = (10 + h)/ x, so we get $\sqrt{3} = (10 + h)/ 10\sqrt{3}$

By further calculation

$10\sqrt{3} \times \sqrt{3} = 10 + h$

30 = 10 + h, h = 30 – 10 = 20

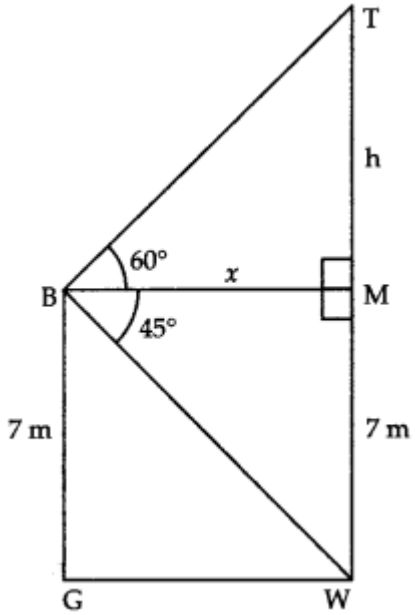
Height of the helicopter from the ground = 10 + 20 = 30 m

2. From the top of a 7 m high building, the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Find the height of the tower.

Answer:

Let BG be building

TW be Tower, then:



BM = x, ∠MBT = 60° ∠MBW = 45°

Draw BM ± TW

In rt. ∠d ΔBMW

tan 45° = WMBM ⇒ 1 = 7x ⇒ x = 7 m

In rt. ∠d ΔTMB

tan 60° = TMBM ⇒ √3 = hx

⇒ h = √3x = 7√3

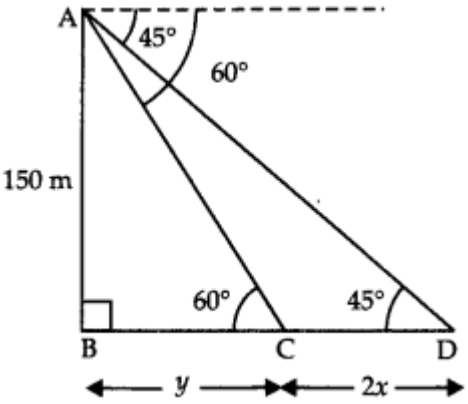
Height of Tower = TW = TM + MW

= (7√3 + 7)m = 7(√3 + 1)m

3. A moving boat is observed from the top of a 150 m high cliff moving away from the cliff. The angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat in m/h. [CBSE 2017]

Answer:

Let C & D be two positions of the boat,& AB be the cliff & let speed of boat be xm/ min.



Let BC = y

∴ CD = 2x (∵ Distance = speed × time)

In ΔABC $\frac{150}{y} = \tan 60°$

$y \Rightarrow 150\sqrt{3} = 50\sqrt{3}$

In ΔABD $\frac{150}{y+2x} = \tan 45°$

$\Rightarrow 150 = 50\sqrt{3} + 2x$

$\Rightarrow x = 25(3 - \sqrt{3})$

∴ Speed = $25(3 - \sqrt{3})$ m/min

= $1500(3 - \sqrt{3})$ m/hr.

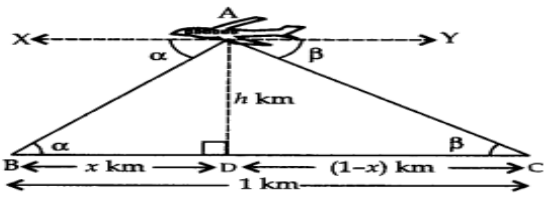
4. From an aeroplane vertically above a straight horizontal plane, the angles of depression of two consecutive kilometres stones on the opposite sides of the aeroplane are found to be α and β. Show that the height of the aeroplane is $\frac{\tan\alpha\tan\beta}{\tan\alpha+\tan\beta}$

Answer:

Let A be the aeroplane and its height be h km further, let B and C be two consecutive kilometres stone so that distance BC = 1 km.

Let BD =x km.

Then DC = (1 – x) km



In right ΔABD

$$\tan\alpha = \frac{AD}{BD} = \frac{h}{x}$$

$$\Rightarrow \quad h = x\tan\alpha$$

$$\Rightarrow \quad x = \frac{h}{\tan\alpha} \qquad \ldots(i)$$

In right ΔADC

$$\tan\beta = \frac{AD}{DC} = \frac{h}{1-x}$$

$$\Rightarrow \quad h = \tan\beta - x\tan\beta \qquad \ldots(ii)$$

Put for x in (ii) from (i), we get

$$h = \tan\beta - \left(\frac{h}{\tan\alpha}\right)\tan\beta$$

$$\Rightarrow \quad h = \frac{\tan\alpha\tan\beta - h\tan\beta}{\tan\alpha}$$

$$\Rightarrow h\tan\alpha = \tan\alpha\tan\beta - h\tan\beta$$

$$\Rightarrow h(\tan\alpha + \tan\beta) = \tan\alpha.\tan\beta$$

$$\Rightarrow \quad h = \frac{\tan\alpha\tan\beta}{\tan\alpha + \tan\beta}$$ **[Hence Proved]**

5. The angle of elevation of a cloud from a point 60 m above the surface of the water of a lake is 30° and the angle of depression of its shadow in water of lake is 60°. Find the height of the cloud from the surface of water.

Answer:

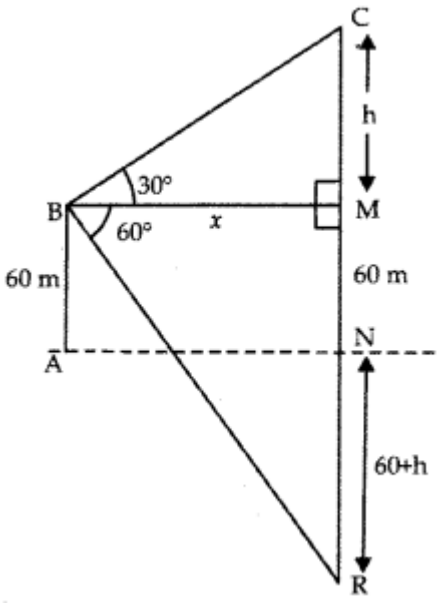
Let C be cloud &B be point 60 m above the surface of water angle of elevation of cloud = ∠MBC = 30°

Angle & Depression of clouds reflection

'R' ∠MBR = 60°

Let BM = x, CM = h, NR = 60 + h,

MR = 60 + 60 + h = 120 + h



In rt. ∠d ΔBMC

$\frac{h}{x} = \tan 30°$

$\Rightarrow x = h\sqrt{3}$

In rt. ∠d ΔBMR

$\frac{60+60+h}{x} = \tan 60°$

$\Rightarrow \frac{120+h}{x} = \sqrt{3}$

$\Rightarrow 120 + h = h\sqrt{3} \times \sqrt{3}$ [using (i)]

$\Rightarrow h = 60$

∵ height of cloud from surface of water = (60 + 60)m = 120 m.

PRACTICE QUESTIONS

1. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff. At a point on the plane 70 metres away from the tower, an observer notices that the angles of elevation of the top and the bottom of the flagstaff are respectively 60° and 45°. Find the height of the flag-staff and that of the tower.

2. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increase from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

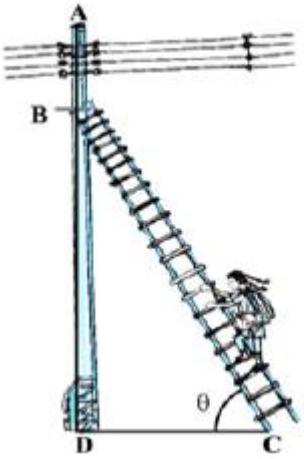
3. An aeroplane is flying at a height of 210 m. Flying at this height at some instant the angles of depression of two points in opposite directions on both the banks of the river are 45º and 60º. Find the width of the river. (Use 3 = 1.73)

4. If the angle of elevation of a cloud from a point h metre above a lake is a and the angle of depression of its reflection in the lake be b, prove that the distance of the cloud from the point of observation is 2 $h\ sec\ \alpha$tan β-tan α.

5. From the top of the tower *h* metre high, the angles of depression of two objects, which are in the line with the foot of the tower are α and β ($\beta > \alpha$). Find the distance between the two objects

Case Study (4 Marks)

1.In a village, group of people complained about an electric fault in their area. On their complaint, an electrician reached village to repair an electric fault on a pole of height 10 m. She needs to reach a point 1.5 m below the top of the pole to undertake the repair work (see the adjoining figure). She used ladder, inclined at an angle to the horizontal such that $\cos\ \theta\ = \frac{\sqrt{3}}{2}$, to reach the required position.



(i)Find the length BD?

(ii) Find the length of ladder.

OR

If the height of pole and distance BD is doubled, then what will be the length of the ladder?

(iii) How far from the foot of the pole should she place the foot of the ladder? [1]

Ans-

(i) Length BD = AD - AB = 10 - 2.5 = 8.5

(ii) The length of ladder BC

In $\triangle$BDC

$\cos\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 30^{o}$

$\sin 30^{o} = \frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{8.5}{BC}$

$\Rightarrow BC = 2 \times 8.5 = 17$ m

OR

If the height of pole and distance BD is doubled, then the length of the ladder is

$\sin 30^{o} = \frac{BD}{BC}$

$\Rightarrow \frac{1}{2} = \frac{17}{BC}$

$\Rightarrow BC = 2 \times 17 = 34$ m

(iii) Distance between foot of ladder and foot of wall CD

In $\triangle$BDC

$\cos 30^{o} = \frac{CD}{BC}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{CD}{17}$

$\Rightarrow CD = 8.5\sqrt{3}$ m

2.Statue of Unity: It is a colossal statue of Indian statesman and independence activist Sardar Vallabh bhai Patel, who was the first Deputy Prime Minister and Home minister of independent India. Patel was highly respected for his leadership in uniting the 562 princely states of India to form the single Union of India. It is located in the state of Gujarat and it is the world's tallest statue.

(i) For a person standing 240 m from the center of the base of the statue, the angle of elevation to the top of the statue is 45°. How tall is the statue? Draw a neat labelled figure to show the above situation diagrammatically. [1]

(ii) For a person standing 240 m from the center of the base of the statue, the angle of elevation to the top of the statue is 45°. How tall is the statue? [1]

(iii) A cop in helicopter near the top of the statue, notices a car accident some distance from the statue. If the angle of depression from the cop's eyes to the distance is 60°, how far away is the accident from the centre of base of the statue? [2]

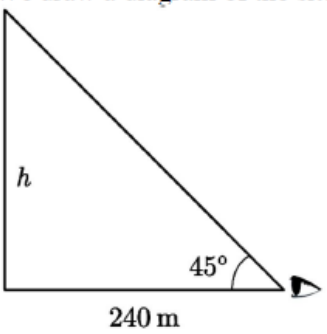
(i)

Ans: Let h be the height of statue. We draw a diagram of the situation as shown below.

Now $\tan 45^\circ = \frac{h}{240}$

$1 = \frac{h}{240}$

$h = 240$ m



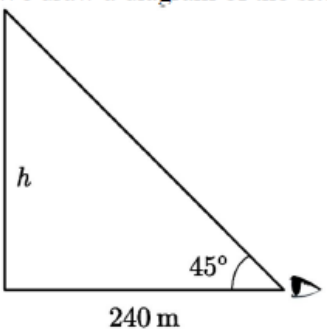
(i)

Ans: Let h be the height of statue. We draw a diagram of the situation as shown below.

Now $\tan 45^\circ = \frac{h}{240}$

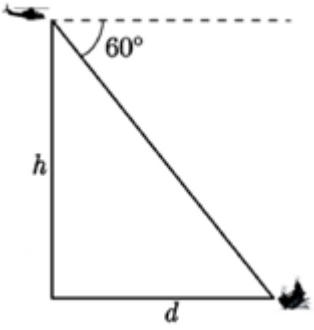
$1 = \frac{h}{240}$

$h = 240$ m



(ii) The statue is 240 mts tall.

(iii) We draw the diagram of the situation as shown below.



Tan 60°= $\frac{h}{d}$

$\sqrt{3} = \frac{240}{d}$ $\Rightarrow$ d = $\frac{240}{3}\sqrt{3}$ mts = $80\sqrt{3}$ mts.

3.A group of Class X students visited Rishikesh in Uttarakhand on a trip. They observed from a point (P) on a river bridge that the angles of depression of opposite banks of the river are 60° and 30° respectively. The height of the bridge is about 18 meters from the river. Based on the above information answer the following questions.

(i)Find the distance PA. [1]

(ii) Find the distance PB. [1]

(iii)Find the width AB of the river. [2]

[OR]

Find the height BQ if the angle of the elevation from P to Q be 30°.

Answer-

$\sin 60° = \frac{PC}{PA}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{18}{PA} \Rightarrow PA = 12\sqrt{3}\ m$

$\sin 30° = \frac{PC}{PB}$

$\Rightarrow \frac{1}{2} = \frac{18}{PB} \Rightarrow PB = 36\ m$

$\tan 60° = \frac{PC}{AC} \Rightarrow \sqrt{3} = \frac{18}{AC} \Rightarrow AC = 6\sqrt{3}\ m$

$\tan 30° = \frac{PC}{CB} \Rightarrow \frac{1}{\sqrt{3}} = \frac{18}{CB} \Rightarrow CB = 18\sqrt{3}\ m$

Width $AB = AC + CB = 6\sqrt{3} + 18\sqrt{3} = 24\sqrt{3}\ m$

[OR]

$RB = PC = 18\ m$ & $PR = CB = 18\sqrt{3}\ m$

$\tan 30° = \frac{QR}{PR} \Rightarrow \frac{1}{\sqrt{3}} = \frac{QR}{18\sqrt{3}} \Rightarrow QR = 18\ m$

$QB = QR + RB = 18 + 18 = 36m$. Hence height BQ is 36m

Practice problems(1 marks)

1. What is the angle of elevation when looking at the top of a tower that is 100 meters away and 50 meters tall?
   A) 30 degrees
   B) 45 degrees
   C) 60 degrees
   D) 90 degrees

2. A ladder is leaning against a wall. If the ladder is 15 meters long and the foot of the ladder is 9 meters away from the base of the wall, what is the height of the wall?
   A) 12 meters
   B) 18 meters
   C) 21 meters
   D) 24 meters

3. In a right triangle, if one acute angle is 30 degrees, what is the measure of the other acute angle?
   A) 45 degrees
   B) 60 degrees
   C) 75 degrees
   D) 90 degrees

4. Two buildings are 200 meters apart. The angle of elevation from the top of one building to the top of the other is 30 degrees. How tall is the taller building?
   A) 100 meters
   B) 115.47 meters
   C) 150 meters
   D) 200 meters

5. A kite is flying at a height of 30 meters. The angle of the string with the ground is 60 degrees. How long is the string?
   A) 15 meters
   B) 30 meters
   C) 60 meters
   D) 90 meters

6. In a right triangle, if one acute angle is 45 degrees, what is the measure of the other acute angle?
   A) 45 degrees
   B) 60 degrees
   C) 75 degrees
   D) 90 degrees

7. Assertion: The angle of elevation of the top of a tower from a point on the ground is always greater than 45 degrees.
   Reasoning: As the distance from the tower increases, the angle of elevation decreases.

A) Both the assertion and reasoning are true, and the reasoning is the correct explanation of the assertion.

B) Both the assertion and reasoning are true, but the reasoning is not the correct explanation of the assertion.

C) The assertion is true, but the reasoning is false.

D) Both the assertion and reasoning are false.

8. Assertion: If the angle of elevation of the sun is 60 degrees, the length of the shadow of a 10-meter-tall pole will be 5 meters.

Reasoning: The length of the shadow is directly proportional to the height of the object and inversely proportional to the angle of elevation of the light source.

A) Both the assertion and reasoning are true, and the reasoning is the correct explanation of the assertion.

B) Both the assertion and reasoning are true, but the reasoning is not the correct explanation of the assertion.

C) The assertion is true, but the reasoning is false.

D) Both the assertion and reasoning are false.

9. Assertion: The length of the shadow cast by a vertical pole on level ground decreases as the angle of elevation of the sun increases.

Reasoning: The length of the shadow is inversely proportional to the angle of elevation of the sun.

A) Both the assertion and reasoning are true, and the reasoning is the correct explanation of the assertion.

B) Both the assertion and reasoning are true, but the reasoning is not the correct explanation of the assertion.

C) The assertion is true, but the reasoning is false.

D) Both the assertion and reasoning are false.

10. Two ships are sailing parallel to each other. The angle of depression from the first ship to the second is 45 degrees, and the ships are 100 meters apart. How far below the first ship is the second ship?

A) 50 meters

B) 70.71 meters

C) 100 meters

D) 141.42 meters

PRACTICE QUESTIONS (2 marks)

1. A ladder, leaning against a wall, makes an angle of 60° with the horizontal. If the foot of the ladder is 2.5 m away from the wall, find the length of the ladder.

2. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a vertical pole to the ground. Find the height of the pole, if the angle made by the rope with the ground level is 30°.

3. The tops of two towers of height x and y, standing on level ground, subtend angles of 30° and 60° respectively at the centre of the line joining their feet, then find x : y.

4. From the top of a vertical tower, the angles of depression of two cars, in the same straight line with the base of the tower, at an instant are found to be 45° and 60°. If the cars are 100 m apart and are on the same side of the tower, find the height of the tower. [Use √3= 1.732]

5. As observed from the top of a 60 m high lighthouse from the sea-level, the angles of depression of two ships are 30° and 45°. If one ship is exactly behind the other on the same side of the light-house, find the distance between the two ships. [Use √3= 1.732]

PRACTICE QUESTIONS(5 marks)

1. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff. At a point on the plane 70 metres away from the tower, an observer notices that the angles of elevation of the top and the bottom of the flagstaff are respectively 60° and 45°. Find the height of the flag-staff and that of the tower.

2. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increase from 30° to 60° as he walks towards the building. Find the distance he walked towards the building.

3. An aeroplane is flying at a height of 210 m. Flying at this height at some instant the angles of depression of two points in opposite directions on both the banks of the river are 45º and 60º. Find the width of the river. (Use 3 = 1.73)

4. If the angle of elevation of a cloud from a point h metre above a lake is a and the angle of depression of its reflection in the lake be b, prove that the distance of the cloud from the point of observation is

2 $h\ sec\ \alpha$tan β-tan α

5. From the top of the tower *h* metre high, the angles of depression of two objects, which are in the line with the foot of the tower are α and β ($\beta > \alpha$). Find the distance between the two objects

PRACTICE QUESTIONS

Case Study (4 Marks)

1.A construction worker is tasked with building a tower. The tower's height and location need to be determined accurately for safety and structural stability.

(i) The construction worker measures the angle of elevation to the top of the tower from a point on the ground, finding it to be 60 degrees. If the worker is standing 50 meters away from the base of the tower, calculate the height of the tower.

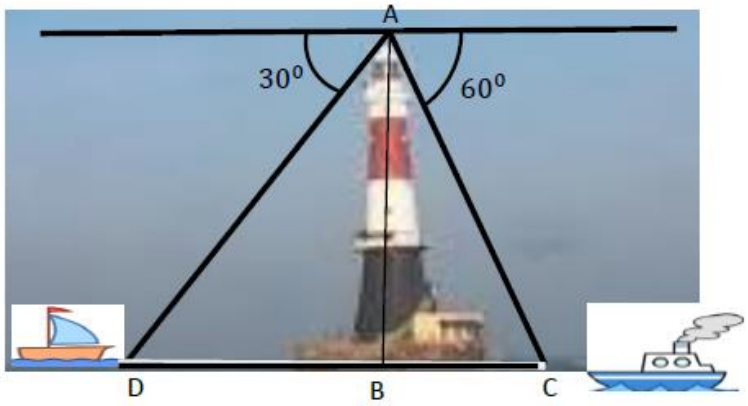
(ii) If the tower's shadow is measured to be 20 meters long at a certain time of day, and the angle of elevation of the sun is 30 degrees, find the height of the tower.

OR

The construction worker needs to ensure that a ladder is long enough to reach the top of the tower. If the ladder's length is 40 meters and it forms a 45-degree angle with the ground when leaning against the tower, find the height of the tower.

(iii) If the construction worker moves 20 meters closer to the tower and measures the new angle of elevation to be 75 degrees, calculate the height of the tower.

2.A lighthouse is a tall tower with light near the top. These are often built on islands, coasts or on cliffs. Lighthouses on water surface act as a navigational aid to the mariners and send warning to boats and ships for dangers. Initially wood, coal would be used as illuminators. Gradually it was replaced by candles, lanterns, electric lights. Nowadays they are run by machines and remote monitoring.
Prongs Reef lighthouse of Mumbai was constructed in 1874-75. It is approximately 40 meters high and its beam can be seen at a distance of 30 kilometres. A ship and a boat are coming towards the lighthouse from opposite directions. Angles of depression of flash light from the lighthouse to the boat and the ship are 30° and 60° respectively.



(i)Which of the two, boat or the ship is nearer to the light house. Find its distance from the light house? [2]

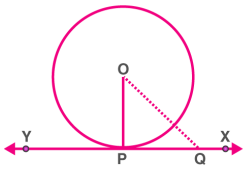
ii) Find the time taken by the boat to reach the light house if it is moving at the rate of 20 km per hour. [2]

CIRCLES

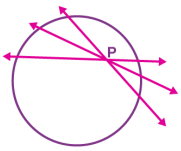
IMPORTANT POINTS:

- A circle is a closed two-dimensional geometrical figure, such that all points on the surface of a circle are equidistant from the point called the "centre". The distance from the centre to any point on the surface of a circle is called "Radius".
- For a circle and a line on a plane, there can be **three** possibilities.
- They can be **non-intersecting.**
- They can have **a single common point:** in this case, the line touches the circle.
- They can have **two common points:** in this case, the line cuts the circle.
- **Tangent:** A **tangent to a circle** is a line that touches the circle at exactly one point. For every point on the circle, there is a unique tangent passing through it.
- **Secant:** A **secant to a circle** is a line that has two points in common with the circle. It cuts the circle at two points, forming a chord of the circle.
- The tangent to a circle can be seen as a special case of the secant when the two endpoints of its corresponding chord coincide.
- For every given **secant** of a circle, there are **exactly two tangents which are parallel** to it and touches the circle at two **diametrically opposite points.**
- **Theorems :**

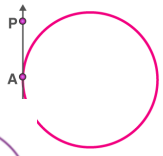
1. **Tangent Perpendicular to the radius at the point of contact:**



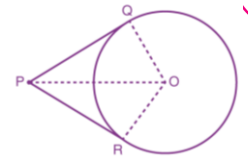
- The theorem states that "The **tangent** to the circle at any point is the **perpendicular to the radius** of the circle that passes through the point of contact".
- Here, O is the centre and OP⊥XY.



2. **The number of tangents drawn from a given point:**
- If the point is in an **interior region of the circle**, any line through that point will be a secant. So, **no tangent** can be drawn to a circle which passes through a point that lies inside it.
- When a point of tangency lies on the circle, there is **exactly one tangent** to a circle that passes through it.

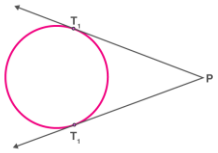


- When the point lies outside of the circle, there are **accurately two tangents** to a circle through it.



3. **Length of tangent:**
- The lengths of tangents drawn from an external point to a circle are equal.
- QP = PR

4. The tangent at any point of a circle is perpendicular to the radius through the point of contact.
5. The lengths of tangents drawn from an external point to a circle are equal.



- **Points to remember:**
- In two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.
- The tangents drawn at the ends of a diameter of a circle are parallel.
- The perpendicular at the point of contact to the tangent to a circle passes through the centre.
- The angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line segment joining the points of contact at the centre.
- The parallelogram circumscribing a circle is a rhombus.

- The opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

MULTIPLE CHOICE QUESTIONS (Solved)

Q1. The given radii of the two concentric circles are 4 cm, and 5 cm, the length of each chord of one circle that is tangent to the other circle is:

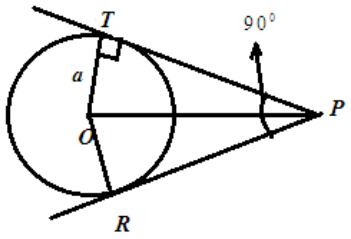
(i) 3 cm b) 6 cm c) 9 cm d) 1 cm

Solution : b) 6 cm
As per the question,
OA = 4cm, OB = 5cm
Also, OA ⊥ BC
Hence, OB2 = OA2 + AB2
52 = 42 + AB2
AB = √(25 – 16) = 3cm
BC = 2AB = 2 × 3cm = 6cm

Q2. If angle between two tangents drawn from a point P to a circle of radius 'a' and centre 'O' is 90°, then OP =

a) 2a√2 b) a√2 c) a/√2 d) 5a√2

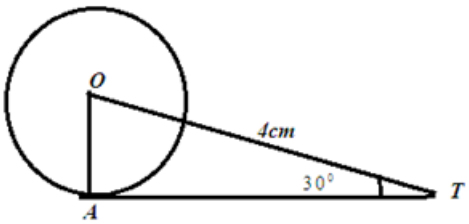
Solution: b) a√2
OT = a (given)
Also line OP bisects the RPT
Therefore,
TPO = RPO = 45º
Also
OT is perpendicular to PT
In right triangle OTP
sin 45° = OT/OP
⇒ 1/√2 = a/OP
⇒ OP = a√2



Q3. In the following figure, AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°. Then AT is equal to :

a) 4 cm b) 2 cm c) 2√3 cm d) 4√3 cm

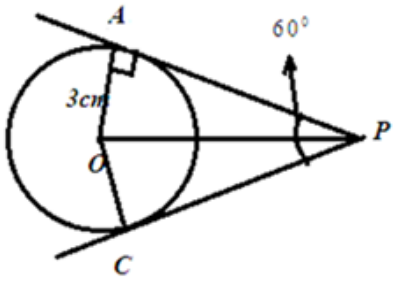
Solution: c) 2√3 cm
We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.
Therefore,
∠OAT = 90°
In triangle OAT
cos30° = AT/OT
√3/2 = AT/4
AT = 2√3 cm



Q4. If two tangents inclined at an angle 60° are drawn to a circle of radius 3 cm, then length of each tangent is equal to:

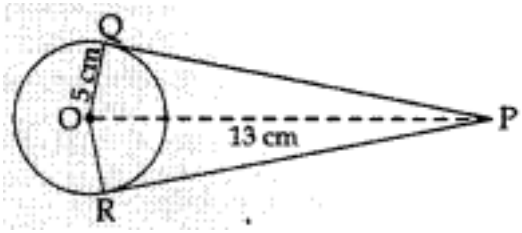
a) $2\sqrt{3}$ cm b) $6\sqrt{3}$ cm c) $3\sqrt{3}$ cm d) 3 cm

Solution: c) $3\sqrt{3}$ cm
Join OA and OP
Also OP is a bisector line of ∠APC
∠APO = ∠CPO = 30°
OA ⊥ AP
Therefore, in triangle OAP
tan30° = OA/AP
$1/\sqrt{3} = 3/AP$
$AP = 3\sqrt{3}$cm



Q5. From a point P which is at a distance of 13 cm from the centre O of a circle of radius 5 cm, the pair of tangents PQ and PR to the circle are drawn. Then the area of the quadrilateral PQOR is:
a) 60 cm² b) 65 cm² c) 30 cm² d) 32.5 cm²

Solution: a) 60 cm²
$OP^2 = OQ^2 + PQ^2$
$169 = 25 + PQ^2$
$PQ^2 = 144$
$PQ = 12$
Area PQOR = area(ΔOPQ) + area(ΔOPR)
$\frac{1}{2} * 12 * 5 + \frac{1}{2} * 12 * 5 = 60$ cm²



VERY SHORT ANSWER QUESTIONS (Solved)

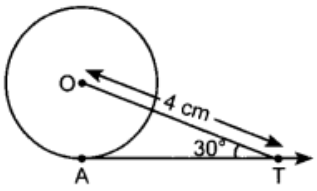
Q1. How many tangents can be drawn from the external point towards the circle?
Solution: Two tangents can be drawn from the external point towards the circle.
Q2. What should be the angle between the two tangents that are drawn at the end of the two radii and are inclined at the angle of 45 degrees?
Solution: The angle between them should be 135 degrees.
Q3. How many tangents could a circle have?
Solution: There can be **infinite** tangents to the circle. A circle is made up of the infinite points that are at an equal distance from the point. As there are infinite points at the circumference of the circle, infinite tangents could be drawn from them.

SHORT ANSWER QUESTIONS (Solved)

Q1. Two parallel lines touch the circle at points A and B respectively. If area of the circle is 25π cm², then find the length of AB.
Solution:
Let radius of circle = R
$\pi R^2 = 25$

R = 5 cm
Distance between two parallel tangents = diameter = 2 * 5 = 10 cm.

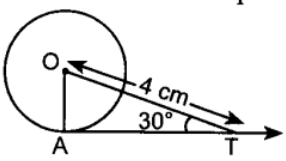
Q2. In figure AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA =30°. Then AT is equal to ?



Solution:

$\angle OAT = 90°$

[∵ Tangent and radius are ⊥ to each other at the point of contact]

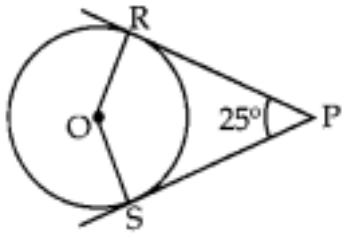


In right-angled ΔOAT,

$$\frac{AT}{OT} = \cos 30°$$

$$\Rightarrow \frac{AT}{4} = \frac{\sqrt{3}}{2} \Rightarrow AT = 2\sqrt{3} \text{ cm.}$$

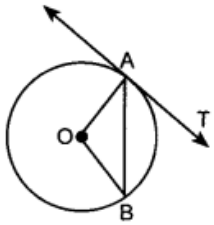
Q3. In the given figure, if ∠RPS = 25°, the value of ∠ROS is



Solution: Since OR ⊥ PR and OS ⊥ PS
∠ORP = ∠OSP = 90°
In ORPS, ∠ROS + ∠ORP + ∠RPS + ∠OSP = 360°
∠ROS + 90° + 25° + 90° = 360°
∠ROS = 360° - 205° = 155°

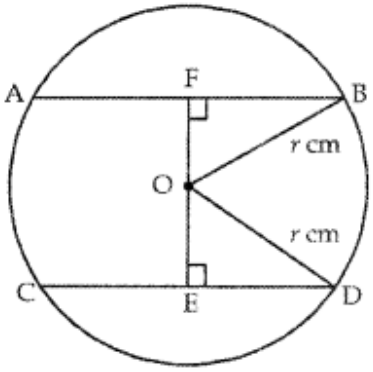
LONG ANSWER QUESTIONS (Solved)

Q1. In figure, O is the centre of a circle, AB is a chord and AT is the tangent at A. If ∠AOB = 100°, then ∠BAT is equal to ?



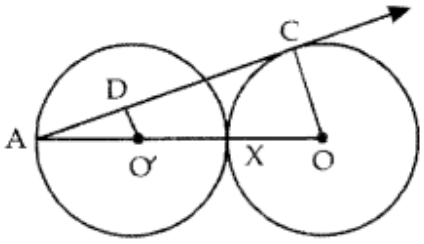
Solution:
∠AOB = 100°
∠OAB = ∠OBA (OA and OB are radii , isosceles triangle)
Now, in ΔAOB,
∠AOB + ∠OAB + ∠OBA = 180° (Angle sum property of Δ)
∠100° + x + x = 180° [Let ∠OAB = ∠OBA = x]
2x = 180° - 100°
2x = 80°
x = 40°
Also, ∠OAB + ∠BAT = 90° [OA is radius and TA is tangent at A]
40° + ∠BAT = 90°
∠BAT = 50°

Q2. AB and CD are two parallels of a circle such that AB = 10 cm and CD = 24 cm. If the chords are on the opposite sides of the centre and the distances between them is 17 cm, find the radius of the circle.

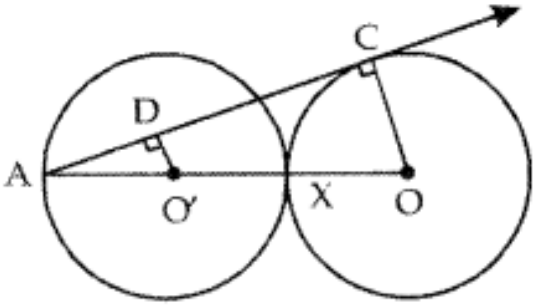


Solution: Let radius of the circle be r cm.
Draw OE ⊥ CD and OF ⊥ AB.
Join OB and OD.
Also, OF = xcm
OE = (17 – x) cm
AB = 10 cm (Given)
∴ FB = 12 AB = 12 × 10 = 5 cm
Similarly, ED = 12 CD = 12 (24) = 12 cm
In right angled ΔOFB,
$OB^2 = OF^2 + FB^2$
$r^2 = x^2 + (5)^2$
$r^2 = x^2 + 25$ ……. (1)
Also, in right angled AOED,
$OD^2 = OE^2 + ED^2$
$r^2 = (17 – x)^2 + (12)^2$
$r^2 = 289 + x^2 – 34x + 144$
$r^2 = x^2 – 34x + 433$ …… (2)
From (1) and (2), we get
$x^2 + 25 = x^2 – 34x + 433$
34x = 408
x = 40834 = 12
Putting the value of x in (1), we get
$r^2 = (12)^2 + 25 = 144 + 25 = (13)^2$
r = 13 cm
Hence, radius of the circle is 13 cm.

Q3. In figure, two equal circles, with centres O and O', touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of $\frac{DO'}{CO}$.



Solution:
Given: Two circles C(O', r) and C(O, r).



AX is diameter of C(O', r) and AC is tangent to C (O, r).O'D ⊥ AC
To find: DO'CO
In ΔAO'D and ΔAOC
∠A = ∠A [Common angle]
∠ADO' = ∠ACO [90° each]
⇒ ΔAO'D ~ ΔAOC
$\Rightarrow \frac{DO'}{CO}=\frac{AD}{AC}=\frac{AO'}{AO}$ ……. (1)
But AO' = r
AO = AO' + O'X + XO
= r + r + r = 3r
(1) $\Rightarrow \frac{DO'}{CO}$=203=$r3r$=13
$\therefore \frac{DO'}{CO}$=13

PRACTICE QUESTIONS (Unsolved)

MULTIPLE CHOICE QUESTIONS

Q1. Choose the correct statement.
a) The center of the circle belongs to the circle.
b) The angle in a semi-circle is a complete angle.
c) Tangents on the diameter endpoints are parallel.
d) Radius equals to twice of the diameter.

Q2. Find the area of the circle if 8 cm is the length of the tangent, 11 cm is the distance between the center of the circle the external point.
a) 100 cm
b) 110 cm
c) 197.14 cm
d) 179.14 cm

Q3. A tangent PQ at the point P of the circle of radius 5 cm meets the line passing through the centre O at the point Q such that OQ = 12 cm. The Length PQ is :
a) 12 cm
b) 13 cm
c) 8.5 cm

d) √119 cm

Q4. The length of tangents drawn from an external point to the circle
a) are equal
b) are not equal
c) sometimes are equal
d) are not defined

Q5. The length of a tangent drawn from a point at a distance of 10 cm of circle is 8 cm. The radius of the circle is
a) 4 cm
b) 5 cm
c) 6 cm
d) 7 cm

VERY SHORT ANSWER QUESTIONS

Q1. A triangle OAB which is an isosceles triangle and AB is tangent to the circle with centre O. Find the measure of ∠OAB.

Q2. Given a right-angled triangle PQR which is right-angled at Q. QR = 12 cm, PQ = 5 cm. The radius of the circle which is inscribed in triangle PQR will be?

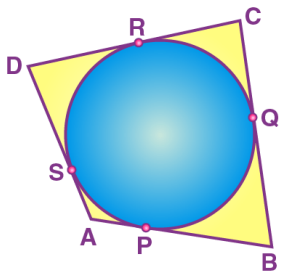
Q3. What is the difference between a secant and a chord of the circle. Is chord also a secant?

SHORT ANSWER QUESTIONS

Q1. From a point Q, the length of the tangent to a circle is 24 cm and the distance of Q from the centre is 25 cm. Find the radius of the circle.

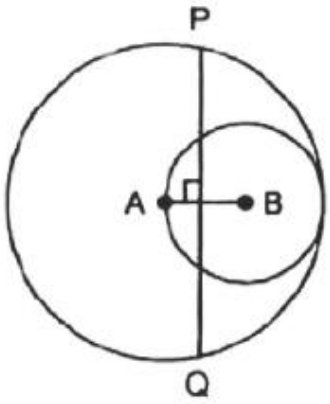
Q2. Prove that the tangents drawn at the ends of a diameter of a circle are parallel.

Q3. A quadrilateral ABCD is drawn to circumscribe a circle as shown in the figure. Prove that AB + CD = AD + BC .



LONG ANSWER QUESTIONS

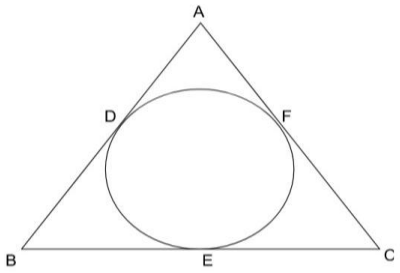
Q1. In the figure, two circles with centres A and B and radii 5 cm and 3 cm touching each other internally. If the perpendicular bisector of segment AB, meets the bigger circle at P and Q, find the length of PQ.



Q2. People of the village want to construct a road nearest to the circular village Parli. The road cannot pass through the village. But the people want the road to be at the shortest distance from the centre of the village. Suppose the road starts from point O which is outside the circular village and touches the boundary of the circular village at point A such that OA = 20 m. And also, the straight distance of the point O from the center C of the village is 25 m.
Find the shortest distance of the road from the centre of the village**.**
If a point is inside the circle, how many tangents can be drawn from that point.

Q3. Varun has been selected by his School to design logo for Sports Day T-shirts for students and staff. The logo design is as given in the figure and he is working on the fonts and different colours according to the theme. In given figure, a circle with centre O is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. The lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively.

a) Find the length of AD
b) Find the length of BE
c) Find the length of CF
d) If radius of the circle is 4 cm, Find the area of ΔOAB
e) Find area of ΔABC.



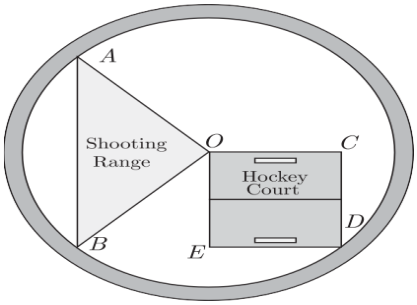
AREA RELATED TO CIRCLES

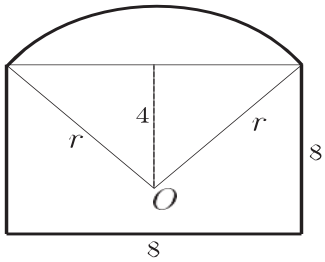
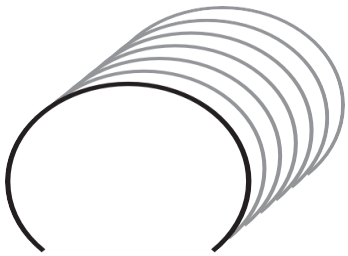
IMPORTANT POINTS:

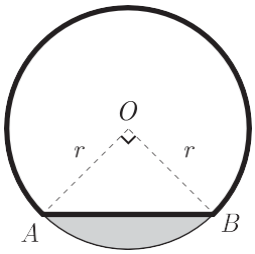
1. Length of an arc of a sector of a circle with radius r and angle with degree measure θ is $\frac{\theta}{360} \times 2\pi r$.

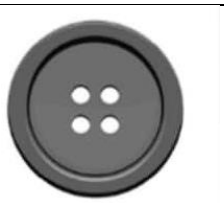
2. Area of a sector of a circle with radius r and angle with degree measure θ is $\frac{\theta}{360} \times \pi r^2$.

3. Area of segment = Area of corresponding sector - Area of corresponding triangle

Q. NO	QUESTIONS	MARKS
1	Jawaharlal Nehru Stadium is a multi-purpose sports stadium and a very popular sports stadium of Delhi. The stadium is a part of the Jawaharlal Nehru sports complex in central Delhi, which also houses the headquarters of the Sports Authority of India, the field arm of the Ministry of Youth Affairs and Sports, and the Indian Olympic Association. It has a capacity to seat 60,000 people. It is the third largest multi-purpose stadium in India. In 2010, the Jawaharlal Nehru Stadium was the main stadium for XIX Commonwealth Games; a major sporting event.   Jawaharlal Nehru stadium is conducting the annual sports competition soon. The curator of the stadium is tasked to figuring out the dimensions for carving out some areas allotted for a hockey court and a shooting range, as shown in the figure below. The shapes of the hockey court and the shooting range are square and triangle respectively. Both of the courts have a common edge that touches the center of stadium. The construction of the shooting range is such that the angle to center is 90c. The radius of the stadium is 200 meters.	
(i)	What is the area allotted for shooting range? a) 12, 600 m^2 b) 22, 000 m^2 c) 20,000 m^2 d) 16, 880 m^2	1
(ii)	What is the area allotted to hockey court?	1

	(a) 12, 600 m2 (b) 22, 000 m2 (c) 20, 000 m2 (d) 16, 880 m2	
(iii)	If the team of the curators managing the stadium, likes to allot space for some more sports, how much area is available to them? (a) 85, 600 m2 (b) 95, 800 m2 (c) 60, 040 m2 (d) 76, 980 m2	1
(iv)	(iv) If the boundaries of the hockey court and shooting range are to be fenced, then what is the required length of the fence? (a) $200(2 + 5\sqrt{3})$ m (b) $200(2 + 3\sqrt{2})$ m (b) (c) $200(2 + 5\sqrt{2})$ m (d) $200(2 + 3\sqrt{3})$ m	1
2	A barn is an agricultural building usually on farms and used for various purposes. A barn refers to structures that house livestock, including cattle and horses, as well as equipment and fodder, and often grain. Ramkaran want to build a barn at his farm. He has made a design for it which is shown above. Here roof is arc of a circle of radius r at centre O.  	
(i)	What is the value of radius of arc ? (a) $4\sqrt{3}$ m (b) $4\sqrt{2}$ m (c) $4\sqrt{5}$ m (d) $2\sqrt{2}$ m	1
(ii)	What is the curved width of roof? (a) $2\pi\sqrt{3}$ m (b) $4\pi\sqrt{2}$ m (c) $2\pi\sqrt{2}$ m (d) $4\pi\sqrt{3}$	1
(iii)	What is area of cross section of barn? (a) $8(6+\pi)$ m^2 (b) $4(6+\pi)$ m^2 (c) $8(3+\pi)$ m^2 (d) $4(3+\pi)$ m^2	1
(iv)	If the length of the barn is 12 meters, what is the curved surface area of roof? a) $32\sqrt{2}\pi$ m^2 b) $16\sqrt{2}\pi$ m^2 c) $48\sqrt{2}\pi$ m^2 d) $24\sqrt{2}\pi$ m^2	1
3	The tunnels are defined as the underground passages that are used for the transportation purposes. These permit the transmission of passengers and freights, or it may be for the transportation of utilities like water, sewage or gas etc. The tunnel engineering is one of the most interesting disciplines in engineering. The work is complex and difficult throughout its course, even though it is interesting.  	

	 Earth is excavated to make a road tunnel. The tunnel is a cylinder of radius 7 m and length 450 m. A level surface is laid inside the tunnel to make road. Figure shows the circular cross - section of the tunnel. The level surface is represented by AB, the center of the circle is O and AOB 90 + = c. The space below AB is filled with rubble (debris from the demolition buildings). Steel girders are erected above the tracks to strengthen the tunnel. The girders are erected at 6 m intervals along the length of the tunnel, with one at each end.	
(i)	What is the cross section area of tunnel before filling debris on ground plane? (a) 154 m^2 (b) 140 m^2 (c) 155 m^2 (d) 145 m^2	1
(ii)	What is the area of cross section of tunnel after filling debris on ground plane? (a) 138 m^2 (b) 140 m^2 (c) 152 m^2 (d) 145 m^2	1
(iii)	What is the length of each girder? (a) 11 m (b) 22 m (c) 33 m (d) 44 m	1
(iv)	How many girders are erected? (a) 76 (b) 75 (c) 74 (d) 73	1
4	Principal of school decided give badges to the students who are chosen for the post of Head boy. Head girl, perfect and vice perfect. Badges are circular in shape with two colour area red and silver as shown in picture. The diameter of region representing red colour is 22 cm and silver colour is filled in 10.5 cm wide ring. 	
(i)	The radius of circle representing red region is: a) 9 cm b) 10 cm c) 11 cm d) 12 cm	1
(ii)	Find the area of red region a) 380.28 cm^2 b) 382.28 cm^2 c) 384.28 cm^2 d) 378.28 cm^2	1
(iii)	Find the radius of the circle formed by combing the red and the silver region a) 20.5 cm b) 21.5 cm c) 22.5 cm d) 23.5 cm	1
(iv)	Find the area of silver region a) 172.50 cm^2 b) 1062.50 cm^2 c) 1172.50 cm^2 d) 1072.50 cm^2	1
5	While doing dusting a maid found a button whose upper face is of black colour, as shown in the figure. The diameter of the each of the smaller identical circles is ¼ of the diameter of the larger circle whose radius is 16 cm. Based on the above information, answer the following questions:	

		
(i)	The area of the each of the smaller circle is a)40.28 cm^2 b) 46.39 cm^2 c) 50.28 cm^2 d) 52.3 cm^2	1
(ii)	The area of the larger circle is a) 804.57 cm^2 b) 704.57 cm^2 c) 855.57 cm^2 d) 990.57 cm^2	1
(iii)	The area of the black colour region is a) 600.45 cm^2 b) 603.45 cm^2 c) 610.45 cm^2 d) 623.45 cm^2	1
(iv)	The area of quadrant of the smaller circle is a) 11.57 cm^2 b) 13.68 cm^2 c) 12 cm^2 d) 12.57 cm^2	1

Q. NO	**ANSWERS**
1(I)	c)20,000 m^2
(ii)	c) 20, 000 m2
(iii)	a) 85, 600 m2
(iv)	a) $200(2+3\sqrt{2})$ m
2(I)	b) $4\sqrt{2}$ m
(ii)	c) $2\pi\sqrt{2}$ m
(iii)	a)$8(6+\pi)$ m^2
(iv)	d) $24\sqrt{2}\pi$ m^2
3(i)	d) 154 m^2
(ii)	b) 140 m^2
(iii)	b)33 m
(iv)	a)76 Girder
4(I)	a)11 Cm
(ii)	a)382.28 Cm^2
(iii)	a)21.5 Cm
(iv)	d) 1072.50 cm^2
5(i)	c) 50.28 cm^2
(ii)	a) 804.57 cm^2
(iii)	b) 603.45 cm^2
(iv)	d) 12.57 cm^2

SURFACE AREAS AND VOLUMES

IMPORTANT POINTS:

Name of the Solid	Curved Surface Area	Total Surface Area	Volume
Cuboid	$2h(l+b)$	$2(lb+bh+hl)$	lbh
Cube	$4a^2$	$6a^2$	a^3
Right Circular Cylinder	$2\pi rh$	$2\pi r(r+h)$	$\pi r^2 h$
Right Circular Cone	πrl	$2\pi r(r+l)$	$\frac{1}{3}\pi r^2 h$
Sphere	–	$4\pi r^2$	$\frac{4}{3}\pi r^2$
Hemisphere	$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^2$

MCQ(with solution)

Choose the correct alternative from the given options:

1. The total surface area 0f the cube is 726 cm^2 . Find the length of its edge.
   (A) 15 cm (B) 12 cm (C) 11 cm (D) 9 cm
2. A cone has a height of 12 cm and a base radius of 3.5 cm, how much ice cream can be put into the cone?
   (A) 154 cm^3 (B) 164 cm^3 (C) 176 cm^3 (D) 184 cm^3
3. Calculate what will be the surface area of a cube whose volume is given as 216 cm^3.
   (A) 316 cm^2 (B) 216 cm^2 (C) 116cm^2 (D) 416 cm^2
4. A right circular cylinder base area is 176 cm^2 and it has a radius of 4cm then find the height of the cylinder?
   (A) 5cm (B) 16cm (C) 8cm (D)7cm
5. In the following questions, A statement of Assertion (A) is followed by a statement of Reason (R). Mark the correct choice as:
   (A) Both A and R are true and R is the correct explanation of A
   (B) Both A and R are true but R is NOT the correct explanation of A.
   (C) A is true but R is false
   (D) A is false but R is True

Assertion (A) : In a right circular cone, the cross-section made by a plane parallel to the base is a circle.

Reason(R): If the volume and the surface area of a solid hemisphere are numerically equal, then the diameter of hemisphere is 9 units.

MCQ(without solution) 1x5=5

Choose the correct alternative from the given options.

6. If the curved surface area of a cylinder in 264 m^2 . Then the ratio of its height to its diameter is
   (A) 3:7 (B) 7:3 (C)1:3 (D)7:1
7. Two cubes of volume 8 cm^3 are joined end to end, then the surface area of resulting cuboid is
   (A) 20cm^2 (B) 40 cm^2 (C) 80 cm^2 (D) 10 cm^2
8. Cylinder, a cone and a hemisphere have same base and same height. The ratio of their volumes is (A)
   (A)1:3:3 (B) 3:3:1 (C) 3:1:3 (D)2:1 :3
9. The lateral surface area of two pillars if height of the pillar is 7m and radius of the base is 1.4m is
   (A)112.3 cm^2 (B) 123.2 cm^2 (C) 345.2 cm^2 (D) 412.2cm^2
10. In this question a statement of Assertion(A) is followed by statement of Reason (R) . Mark the correct choice as:

A) Both assertion (A) and reason (R) are true and reason(R) is the correct explanation of assertion (A)
(B)Both assertion (A) and reason (R) are true but reason (R) is not the correct explanation of assertion (A)
(C) Assertion (A) is true but reason (R) is false
(D) Assertion (A) is false but reason(R) is true

Assertion(A): The area of a sector to a circle of radius 1.4cm and central angle 90^0 is 1.54 cm^2.
Reason (R): If central angle is 60^0, then area of a sector is 1.03 cm^2 .

(SA- with solution) 2x3=6

11. . Volume and surface area of a solid hemisphere are numerically equal. What is diameter of hemisphere?
12. Two cubes each of volume 64 cm^3 are joined end to end. Find the surface of the resulting cuboid.
13. If the ratio of volume of two spheres is 1:8, then find the ratio of their surface area.

(SA – without solution) 2x3=6

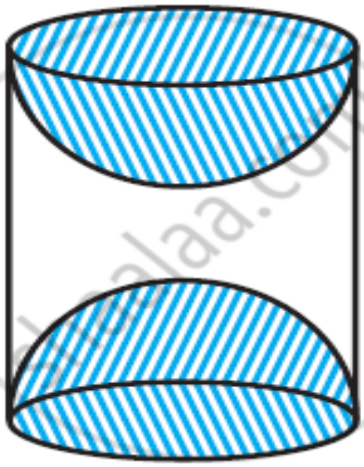
14. From a solid cube of side 14 cm, a sphere of maximum diameter is carved out. Find the radius of the sphere.
15. Find the volume of the largest right circular cone that can be cut- out from a cube of edge 4.2 cm.
16. If the perimeter of a protector is 72 cm, calculate its area

(SA- with solution) 3x3=9

17. Due to heavy floods in a state, thousands were rendered homeless. 50 schools collectively offered to the state government to provide place and the canvas for 1500v tents to be fixed by the government and decided to share their whole expenditure equally. The lower part of each tent is cylindrical of base radius 2.8m and height 3.5m, with conical upper part of same base radius but of height 2.1 m. If the canvas used to make the tents cost Rs.120 per m^2, then find the amount shared by each school to set up the tents.

18 .A solid is the shape of cone standing on a hemisphere with both their radii being equal to 1 cm and the height of the cone is equal to its radius. Find the volume of the solid.

19. A wooden article was made by scooping out a hemisphere from each end of a solid cylinder, as shown in adjacent figure . If the height of the cylinder is 10cm and its base is of radius 3.5 cm, then find the total surface area of the article.



(SA- without solution) **3x3=9**

20. An hour glass is made using identical double glass cones of diameter 10cm each. If total height is 24 cm, then find the surface area of the glass used in making it.
21. A solid is composed of a cylinder with hemispherical ends. If the whole length of the solid is 108 cm and the diameter of the hemispherical ends is 36cm, find the cost of polishing the surface of the solid at the rate of 7 paise per sq cm.
22. $A\ conical\ hole\ is\ drilled\ \ in\ a\ circular\ cylinder\ of\ height\ 15\ cm\ and\ radius$ 8cm, which has same height and same base radius. Find the total surface area after drilling of the cone [take π=3.14]

ANSWER KEY

1.C (11cm) 2. A(154 cm) 3 B(216 cm) 4 D(7 cm) 5 0ption (B) is correct

11. Volume of hemisphere=S A of solid hemisphere $>$ $2/3\pi r^3 = 3\pi r^2$

So, r=9/2 $\therefore$ diameter=2r= 9 units.

12. Let the edge of each cube be a cm . now a^3= 64 , so a=8 when two cubes are joined, length of cuboid 2x8=16 cm breadth=8 cm and height= 8 cm, then surface area of cuboid=2(16x8 +8x8 +16x8)= 640 cm^2

13. Ratio=$\frac{volume\ of\ 1st\ sphere}{volume\ of\ 2nd\ sphere}$ = $4/3\pi\ R^3 \div 4/3\pi r^3$ >R/r=1/2

17 Canvas required for one tent= Curved surface area of cylinder + Curved surface area of cone. So canvas required for 1500 tents = 1500x canvas required for one tent = Rs,120x canvas required for 1500 tents . Hence, amount shared by the school= cost of the canvas/50=Rs.332640

18. Given, solid is a combination of a cone and hemisphere. Also , radius of cone ,r= Radius of the hemisphere=1cm, Height of the cone ,h=1 cm

So, Required volume of the solid = Volume of the cone+ volume of the hemisphere=$1/3\pi r^2h + 2/3\pi r^3$ $=1/3\ \pi(1)^2 +2/3\pi\ (1)^3= \pi$

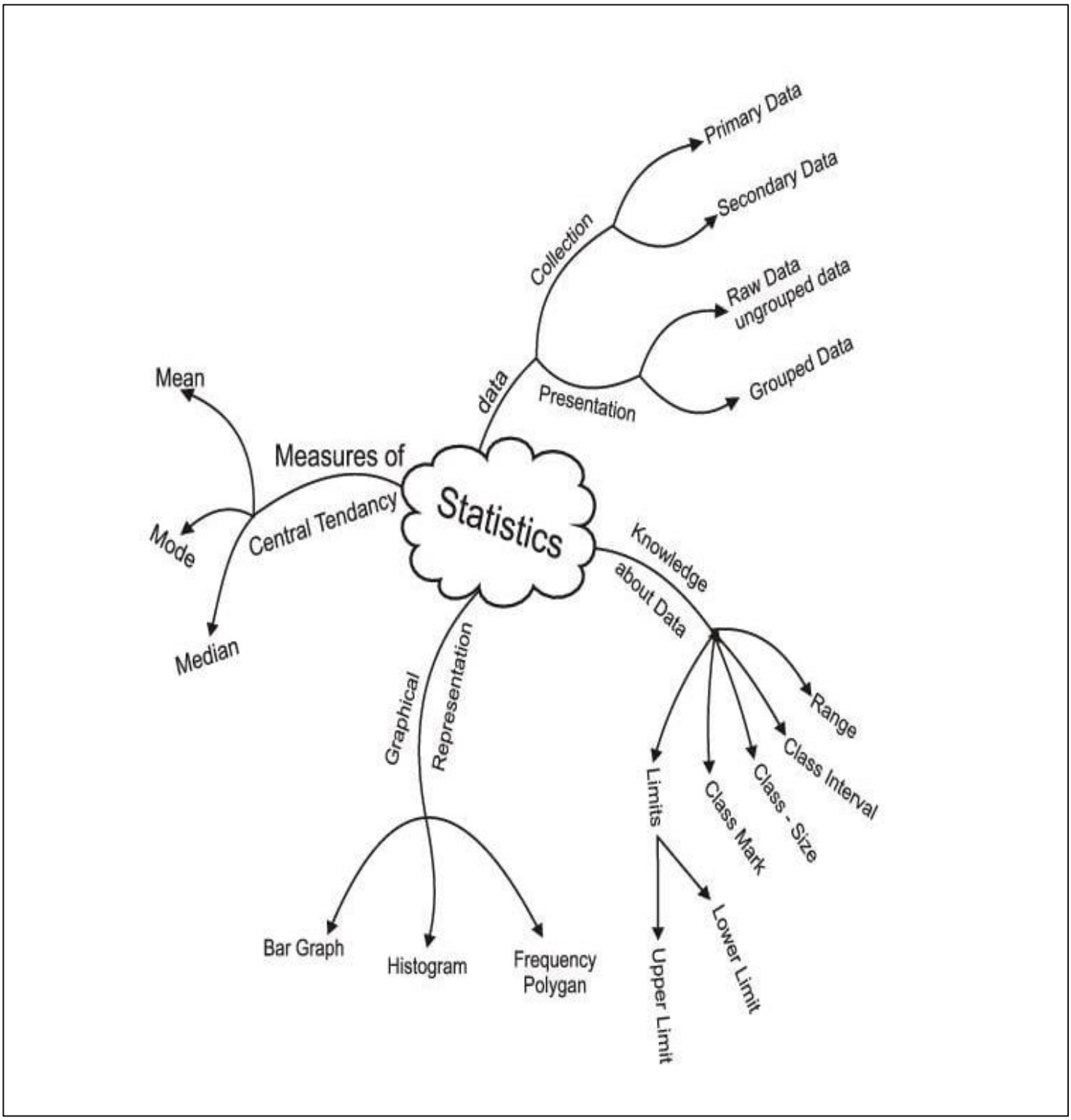
19. Given : wooden article is a combination of a cylinder and two hemisphere. Here height of the cylinder ,h=10cmRadius of the base of the cylinder= Radius of hemisphere , r=3.5cm Now ,required TSA of the wooden article=2Xcsa of cylinder= $2x(2\pi r^2) + 2\pi rh= 2x22/7x3.5x(2x3.5 +10)=374\ cm^2$

20.Ans $180\pi\ cm^2$ 21.Ans Rs. 855.36 22. Ans. 1381.6 cm^2.

STATISTICS

IMPORTANT POINTS:

Mind Map



Key Points

1. Data Types:
    - Discrete Data: Data that can only take certain values (usually whole numbers) such as the number of students in a class.
    - Continuous Data: Data that can take any value within a range such as height or weight.

2. Collection of Data:
    - Methods of data collection, including surveys, experiments, and observational studies.
    - Primary Data: Data collected firsthand by the researcher.
    - Secondary Data: Data collected by someone else for another purpose.
3. Organization of Data:
    - Tabulation: Arranging data in rows and columns.
    - Frequency Distribution: Organization of data according to the frequency of occurrence of each value.
    - Grouped Frequency Distribution: Data organized into intervals or classes.
4. Measures of Central Tendency:
    - Mean: The average of a set of data.
    - Median: The middle value when the data is arranged in ascending or descending order.
    - Mode: The value that occurs most frequently in a data set.
    - 3Median = Mode + 2Mean

(a) Mean of Grouped Data

(i) (To find the mean of grouped data, it is assumed that the frequency of each class interval is centred around its mid-point.

(ii)Direct Method Mean,

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

where the xi (class mark) is the mid-point of the ith class interval and f i is the corresponding frequency

(iii) Assumed Mean Method,

$$\text{Mean } (\bar{x}) = a + \frac{f_i d_i}{f_i},$$

a is the assumed mean and di = xi – a are the deviations of xi from a for each i.

(iv) Step-deviation Method,

$$\text{Mean } (\bar{x}) = a + h \frac{f_i u_i}{f_i},$$

where a is the assumed mean, h is the class size and u = (x-a)/h

(v) If the class sizes are unequal, the formula in (iv) can still be applied by taking h to be a suitable divisor of all the di ’s.

(b) Mode of Grouped Data

(i) In a grouped frequency distribution, it is not possible to determine the mode by looking at the frequencies. To find the mode of grouped data, locate the class with the maximum frequency. This class is known as the modal class. The mode of the data is a value inside the modal class.

(ii) Mode of the grouped data can be calculated by using the formula

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

where l is the lower limit of the modal class, h is the size of the class, f1 is frequency of the modal class and f0 and f2 are the frequencies of the classes preceding and succeeding the modal class, respectively.
(c) Median of Grouped Data
(i) Cumulative frequency table – the less than type and the more than type of the grouped frequency distribution.
(ii) If n is the total number of observations, locate the class whose cumulative frequency is greater than (and nearest to) class. 2 n . This class is called the median class.
(iii) Median of the grouped data can be calculated by using the formula

$$\text{Median} = l + \left(\frac{\frac{N}{2} - c.f.}{f} \right) \times h$$

where l is the lower limit of the median class, n is the number of observations, h is the class size, cf is the cumulative frequency of the class preceding the median class and f is the frequency of the median class.

Solved Questions

Multiple choice questions 5*1 = 5Marks

1. In a class of 20 students, the following scores (out of 100) were obtained in a Mathematics test: 20,30,40,50,60,70,80,90,100,30,40,50,60,70,80,90,100,30,40,5020,30,40,50,60,70,80,90,100,30,40,50,60,70,80, 90,100,30,40,50
What is the median score?
a) 50
b) 55
c) 60
d) 65
Answer: a) 50
Explanation: The data needs to be arranged in ascending order first:
20,30,30,30,40,40,40,50,50,50,60,60,70,70,80,80,90,90,100,100
Since there are 20 values, the median is the average of the 10th and 11th values, which are both 50.

2. In a survey of 50 people, 15 people preferred Brand A, 20 people preferred Brand B, and the rest preferred Brand C. What is the mode of the survey results?
a) Brand A
b) Brand B
c) Brand C
d) No mode
Answer: d) No mode
Explanation: Since no brand preference occurs more frequently than the others, there is no mode.
3. The ages (in years) of 10 students are: 12, 14, 12, 15, 13, 12, 16, 14, 13, 14. What is the mean age of the students?
a) 13
b) 13.5
c) 14
d) 14.5
Answer: b) 13.5
Explanation: Sum of ages = 12+14+12+15+13+12+16+14+13+14=13512+14+12+15+13+12+16+14+13+14=135
Mean age = 135/10=13.5

4. The heights (in cm) of 10 students are: 150, 155, 160, 165, 170, 170, 175, 180, 185, 190. What is the median height of the students?
a) 165 cm
b) 170 cm
c) 172.5 cm
d) 175 cm
Answer: b) 170 cm
Explanation: Since there are 10 values, the median is the average of the 5th and 6th values, which are both 170 cm.
5. Assertion: the mode of the call received on 7 consecutive day 11,13,13,17,19,23,25 is 13.
Reason: Mode is the value that appears most frequent;
a)Both Assertion and Reason are correct and reason is correct explanation for the assertion .
b)Both Assertion and Reason are false but reason is not correct explanation for assertion.
c)Assertion is correct but reason is false.
d)Both Assertion and reason are false.
Answer : a) Both Assertion and Reason are correct and reason is correct explanation for the assertion .

Section B: Short Answer Type Questions (2marks)

6. The following table shows the number of hours spent by 20 students on a project:

Hours Spent (in hours)	Number of Students
5	4
10	6
15	5
20	3
25	2

Calculate the mean number of hours spent by students on the project.
Solution:
Using the formula for the mean:

$$\text{Mean} = \frac{\text{Sum of all values}}{\text{Total number of values}}$$

$$\text{Mean} = \frac{(5\times4)+(10\times6)+(15\times5)+(20\times3)+(25\times2)}{20}$$

$$\text{Mean} = \frac{20+60+75+60+50}{20} = 265/20 = 13.25 \text{ hours}$$

Therefore, the mean number of hours spent by students on the project is 13.25 hours.
7. The following data represents the number of goals scored by a football team in 10 matches:
3, 1, 2, 4, 2, 3, 0, 1, 2, 3. Find the mode of the data.
Solution: Mode is the value that appears most frequently in the dataset. Here, the number of goals scored that appears most frequently is 2. Therefore, the mode of the data is 2.

8. Find the mode of the following frequency distribution:

Class	25−30	30−35	35−40	40−45	45−50	50−55

Interval:						
Frequency:	25	34	50	42	38	14

Soln:

Class Interval	Frequency
25–30	25
30–35	34
35–40	50
40–45	42
45–50	38
50–55	14

Here, maximum frequency is 50. So, 35 - 40 will be the modal class.

$$l = 35, f_0 = 34, f_1 = 50, f_2 = 42 \text{ and } h = 5$$

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 35 + \left(\frac{50 - 34}{2 \times 50 - 34 - 42} \right) \times 5$$

$$= 35 + \left(\frac{16}{100 - 76} \right) \times 5$$

$$= 35 + \frac{16}{24} \times 5$$

$$= 35 + \frac{80}{24}$$

$$= 35 + 3.33$$

$$= 38.33$$

Section C: Short Answer Type Questions (3 marks)

10. The marks obtained by 100 students in an examination are given below:

Marks	Number of Students
30–35	14
35–40	16
40–45	28
45–50	23
50–55	18
55–60	8
60–65	3

Find the mean marks of the students.

Solution.

Class Interval (Marks)	No. of Students (fi)	xi	fi xi
30-35	14	32.5	455
35-40	16	37.5	600
40-45	28	42.5	1190
45-50	23	47.5	1092.5
50-55	18	52.5	945
55-60	8	57.5	460
60-65	3	62.5	187.
	Sfi = 110		Sfi xi = 4930

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

= 4930/110

= 44.81

10. Find the mode of the following frequency distribution.

Class	Frequency
0−10	8
10−20	10
20−30	10
30−40	16
40−50	12
50−60	6
60−70	7

Solution. The given frequency distribution table is above

Here, the maximum class frequency is 16

Modal class = 30 - 40

lower limit (l) of modal class = 30

Class size (h) = 10

Frequency (f1) of the modal class = 16

Frequency (f0) of preceding class = 10

Frequency (f2) of succeeding class = 12

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2} \right) \times h$$

$$= 30 + \left(\frac{16 - 10}{32 - 10 - 12} \right) \times 10$$

$$= 30 + \frac{6}{32 - 22} \times 10$$

$$= 30 + \frac{6}{10} \times 10$$

$$= 30 + 6 = 36$$

Hence, Mode = 36.

11. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	Frequency
0 – 20	12
20 – 40	15
40 – 60	32
60 – 80	K
80 – 100	13

Solution. Given, Median = 53

Class	Frequency fi	Mid value xi	fixi
0 – 20	12	10	120
20 – 40	15	30	450
40 – 60	32	50	1600
60 – 80	k	70	70k
80 – 100	13	90	1170
	72=k		3340 + 70k

$$\text{Mean} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

$$53 = \frac{3340 + 70k}{72 + k}$$

53(72 + k) = 3340 + 70k

3816 + 53k = 3340 + 70k

K = 28, Hence k = 28.

Section D: Long Answer Type Question (5 marks)

12. If the median of the following frequency distribution is 32.5. Find the values of f1 and f2.

Class	Frequency
0-10	f1
10-20	5
20-30	9
30-40	12
40-50	f2
50-60	3
60-70	2

Solution. Median = 32.5

Class	Frequency	Cumulative Frequency
0-10	f1	f1
10-20	5	f1 + 5
20-30	9	f1 + 14
30-40	12	f1 + 26
40-50	f2	f1 + f2 + 26
50-60	3	f1 + f2 + 29
60-70	2	f1 + f2 + f3

Total Frequency = 40

Therefore f1 + f2 + 31 = 40

Or f1 + f2 = 9 ……..(i)

Also n/2 = 40/2 = 20

Median 32.5 (given)

Which lies in the class interval (30 – 40)

Since Median = 30 – 40

So, $l = 30, f = 12, C.f. = f_1 + 14, h = 10$

$$\text{So,} \quad \text{Median} = l + \left[\frac{\frac{n}{2} - C.f.}{f}\right] \times h$$

$$32.5 = 30 + \left[\frac{20 - (f_1 + 14)}{12}\right] \times 10$$

$$32.5 = 30 + \left(\frac{6 - f_1}{6}\right) \times 5$$

$$\text{or} \quad 2.5 = \frac{5}{6}(6 - f_1)$$

$$\text{or} \quad \frac{2.5 \times 6}{5} = 6 - f_1$$

$$\text{or} \quad 6 - f_1 = 3 \Rightarrow f_1 = 3$$

From equation (i), we get

$$f_2 = 6$$

$$\therefore \quad f_1 = 3, \ f_2 = 6$$

Section E: Case Study (4 marks)

13. Life insurance is a contract between an insurance policy holder and an insurer or assurer, where the insurer promises to pay a designated beneficiary a sum of money upon the death of an insured person (often the policy holder). Depending on the contract, other events such as terminal illness or critical illness can also trigger payment. The policy holder typically pays a premium, either regularly or as one lump sum.

SBI life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate the median age, if policies are given only to persons having age 18 years onwards but less than 60 years.

Age (in years)	Below 20	Below 25	Below 30	Below 35	Below 40	Below 45	Below 50	Below 55	Below 60
Number of policy holders	2	6	24	45	78	89	92	98	100

(i)What is the median value of age ?

(ii)What will be the upper limit of the modal class? What is the mode value of age ?

(iii) Find the mean value of age using empirical relation.

Solution. The given table is cumulative frequency distribution. We write the frequency distribution as given below:

Class interval	Cumulative Frequency	Frequency
15-20	2	2-0 = 2
20-25	6	6-2 = 4
25-30	24	24-6 = 18
30-35	45	45-24 = 21
35-40	78	78-45 = 33
40-45	89	89-78 = 11
45-50	92	92-89 = 3
50-55	98	98-92 = 6
55-60	100	100-98 = 2

We have, Σf_i = N = 100

(i) Cumulative frequency just greater than N/2 = 100/2 = 50 is 78 and the corresponding class is 35-40. Thus median class is 35-40.

Now, l 35, N/2=50, F =c.f= 45, f = 33 and h = 5

$$\text{Median} = l + \left[\frac{\frac{n}{2} - C.f.}{f}\right] \times h$$

Median,

Md = 35+[(50-45)/33]*5

= 35+ 25/33

= 35 = .76 = 35.76 years

Thus, the median age 35.76 years.

(ii) Class 35-40 has the maximum frequency 78, therefore this is model class.

Now l = 35, f1 = 33, f2 = 11, fo = 21, h = 5

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

= 35+ [(33-21)/(66-21-11)] *5

= 35 + 12*5/33

= 35 + 1.82 + 36.82 years

(iii) Empirical relation,

3Md = Mo +2M

3*35.76 = 36.82 +2M

107.28 = 36.28 +2M

2M = 107.28 – 36.82

M = 35.23 years.

Unsolved Questions

Multiple choice questions 5*1 = 5 marks

1. In a frequency distribution, the mid value of a class is 10 and the width of the class is 6. The lower limit of the class is

(a) 6 (b) 7

(c) 8 (d) 12

2. Mode of the following grouped frequency distribution is

Class	Frequency
3-6	2
6-9	5
9-12	10
12-15	23
15-18	21
18-21	12
21-24	03

(a) 13.6 (b) 15.6 (c) 14.6 (d) 16.6

3. If median is 137 and mean is 137.05, then the value of mode is

(a) 156.90 (b) 136.90

(c) 186.90 (d) 206.90

4. While computing mean of grouped data, we assume that the frequencies are

(a) evenly distributed over all the classes

(b) centred at the class marks of the classes

(c) centred at the upper limits of the classes

(d) centred at the lower limits of the classes

5. Assertion: The mean is a measure of central tendency.

Reason: The mean is calculated by adding up all the values in a dataset and dividing by the total number of values.

A) Both the assertion and reason are correct, and the reason is the correct explanation of the assertion.

B) Both the assertion and reason are correct, but the reason is not the correct explanation of the assertion.

C) The assertion is correct, but the reason is incorrect.

D) Both the assertion and reason are incorrect.

Section B: Short Answer Questions (2marks)

6. Write the median class of the following distribution:

Classes	0 -10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	4	4	8	10	12	8	4

7. The mean of the following frequency distribution is 25. Find the value of p.

Class interval	0-10	10-20	20-30	30-40	40-50
Frequency	4	6	10	6	p

8. Convert the following data into 'more than type' distribution:

Class	50-55	55-60	60-65	65-70	70-75	75-80
Frequency	2	8	12	24	38	16

Section C: Short Answer Questions (3 marks)

9. A group of students conducted a survey of their locality to collect the data regarding number of plants and recorded it in the following table:

Number of plants	0-3	3-6	6-9	9-12	12-15
Number of houses	2	4	5	1	2

Find the mode for the above data.

10. The mean of the following distribution is 53. Find the missing frequency p :

Class	0-20	20-40	40-60	60-80	80-100
Frequency	12	15	32	p	13

11. The mean of n observations is x, if the first term is increased by 1, second by 2 and so on. What will be the new mean?

Section D: Long Answer Question (5 marks)

12. On the sports day of a school, 300 students participated. Their ages are given in the following distribution:

Age (in years)	5-7	7-9	9-11	11-13	13-15	15-17	17-19
Number of students	67	33	41	95	36	13	15

Find the mean and mode of the data.

Section E: Case Study (4 marks)

13. 100 Metres Race: The 100 metres is a sprint race in track and field competitions. The shortest common outdoor running distance, it is one of the most popular and prestigious events in the sport of athletics. It has been contested at the summer Olympics since 1896 for men and since 1928 for women. The World Championships 100 metres has been contested since 1983. The reigning 100 m Olympic or world champion is often named "the fastest man or woman in the world".

A stopwatch was used to find the time that it took a group of students to run 100 m.

Time (in sec)	0-20	20-40	40-60	60-80	80-100
No. of students	8	10	13	6	3

Based on the above information, answer the followingquestions.

(i) Estimate the mean time taken by a student to finish the race.

(ii) What will be the upper limit of the modal class ?

(iii) What is the sum of lower limits of median class and modal class ?

(iv) How many students finished the race within 1minute?

ANSWERS

1. b
2. c
3. b
4. b
5. a
6. 20-20
7. 4
8.

Class	Frequency
More than 50	100
More than 55	98
More than 60	90
More than 65	78
More than 70	54
More than 16	16

9. 6.6
10. 28
11. (n+1)/2
12. mean 10.66 , mode 11.95
13. (i) 43 sec (ii) 60 (iii) 80 (iv) 31

PROBABILITY

IMPORTANT POINTS:

PROBABILITY:

Probability is a concept which numerically measures the degree of certainty of the occurrence of events. The chance that an event will or will not occur is

expressed on a scale ranging from 0 to 1.

It can also be represented as a percentage.

EXPERIMENT:

An operation which can produce some well-defined outcomes is called an experiment.

RANDOM EXPERIMENT:

An experiment in which all possible outcomes are known, and the exact outcome cannot be predicted in advance, is called a random experiment.

Event and outcome

An Outcome is a result of a random experiment. For example, when we roll a dice getting six is an outcome

An Event is a set of outcomes. For example, when we roll a dice the probability of getting a number less than five is an event.

Note:

An Event can have a single outcome.

EQUALLY LIKELY EVENTS :

A given number of events are said to be equally likely if none of them is expected to occur in preference to the others.

Experimental Probability

Experimental probability can be applied to any event associated with an experiment that is

repeated a large number of times.

A **trial** is when the experiment is performed once. It is also known as **empirical probability**.

Experimental or empirical probability:

P(E) = Number of trials where the event occurred / Total number of trials

Theoretical Probability

P(E) =Number of outcomes favorable to E /Number of all possible outcomes of the experiment.

Here we assume that the outcomes of the experiment are equally likely.

Elementary Event

An event having only one outcome of the experiment is called an elementary event.

Example: Take the experiment of tossing a coin n number of times. One trial of this

experiment has two possible outcomes: Heads(H) or Tails(T). So for an individual toss, it has only one outcome, i.e Heads or Tails.

Probability of occurrence of an event E, denoted by P(E) is defined as:

P(E) = no of outcomes favourable to E /total no of possible outcomes.

Sum of Probabilities

The sum of the probabilities of all the elementary events of an experiment is one.

Example: take the coin tossing experiment. P(Heads) + P(Tails) = 1/2+1/2 = 1.

SURE EVENT:

The sure event is defined as an event which always happens. Hence, the probability of a sure event is always 1. For example, in single throw of a die, P(getting a number ≤ 7) = = 6/6=1.

IMPOSSIBLE EVENT:

An event which is impossible to occur, is called an impossible event. The probability of impossible event is always zero. For example, in single throw of a die, P(getting 7) = 0/6= 0.

COMPLEMENTARY EVENT:

Let E be an event and (not E) be an event which occurs only when E does not occur. We denote (not E) by E', or , called complement of event E. The event (not E) is called the complementary event of E. P(E) + P(not E) = 1.
$\therefore$ P(E) = 1 − P(not E).

FACTS ABOUT TOSSING COINS:

When we toss a coin, all the outcomes are H, T. So, number of all outcomes = 2

When we toss two coins, all the outcomes are (H,H), (H,T), (T,H), (T,T). So number of all outcomes are 2×2=4

When we toss three coins, all the outcomes are

(H,H,H), (H,H,T), (H,T,H), (H,T,T), (T,H,H), (T,H,T), (T,T,H), (T,T,T). So number of all outcomes are 2×2×2=8

FACTS ABOUT ROLLING DICES:

When we roll one dice, the number of all outcomes= 6. Outcomes are 1,2,3,4,5,6.

When we roll two dices, the number of all outcomes=6x6=36

Outcomes are
(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(2,1),(2,2),(2,3),(2,4),(2,5),(2,6),(3,1),(3,2),(3,3),(3,4),(3,5),(3,6),(4,1),(4,2),(4,3),(4,4),(4,5),(4,6),(5,1),(5,2),(5,3),(5,4),(5,5),(5,6).

FACTS ABOUT PLAYING CARDS:

Let us take an example related to playing cards. Have you seen a deck of

playing cards? It consists of 52 cards which are divided into 4 suits of 13 cards each—

spades (♠), hearts (♥), diamonds (♦) and clubs (♣). Clubs and spades are of black

colour, while hearts and diamonds are of red colour. The cards in each suit are ace,

king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2. Kings, queens and jacks are called face cards.

Q.N	SECTION - A MULTIPLE CHOICE QUESTIONS (1 MARK EACH) (SOLVED)	MARKS
1	Two coins are tossed simultaneously. The Probability of having exactly one head is (a) 1/4 (b) 1/2 (c) 3/4 (d) none of these Ans :(c) Total no. of outcomes = 4(HT,TH,HH,TT) No. of outcomes containing exactly one head = 2(HT,TH) Probability of outcomes of getting exactly one head= 2/4=1/2	1
2	Probability that cannot exist among the following: (a) 2/3 (b) -2.6 (c) 25% (d) 0.7 Ans: (b) -2.6 Explanation: $0 \leq P(E) \leq 1$, for any event E.Hence, it cannot be negative.	1
3	The probability that the drawn card from a pack of 52 cards is neither an ace nor a spade is (a)9/13 (b)35/52 (c)19/26 (c)10/13 Ans: The total number of cards = 52 The number of aces in a pack of cards = 4 The number of spades in a pack of cards = 13 The total number of aces and spades =4+13=17 As one of the spades is an ace so one card will be in common therefore the total number of cards will be one less than the sum of spades and aces. = 4 + 13 – 1= 16 The total number of cards in the pack that will neither be an ace or a spade = 52 – 16= 36 The probability of a card in the pack neither being an ace or a spade = Cards in the pack that are neither ace nor spade / total number of cards =36/52=9/13 Therefore, the probability that the drawn card from a pack of 52 cards is neither an ace nor a spade is 9/13.	1

4	When a die is thrown once, the probability of getting an even number less than four is (a)1/4 (b)0 (c)1/2 (d)1/6 Ans: In a throw of a dice, Possible outcomes ={1,2,3,4,5,6} Total number of possible outcomes =6 Now, Favorable outcomes =Even numbers ={2,4,6} Number of favorable outcomes =3 Probability is given by, $P(A)$=no. of possible outcomes/No. of total outcomes P(an even number) =3/6=1/2 Hence option : (c) 1/2	1
5	The question consist of two statement – Assertion (A) and Reason. Answer the question selecting the appropriate option given below: (a) Both A and R are true is the correct exploration for A. (b) Both A and R are true and R is not the correct exploration for A. (c) A is true but R is false. (d) A is false but R is true. Assertion (A) :The probability of getting a prime number. When a die is thrown once is $\frac{2}{3}$. Reason (R) :Prime numbers on a die are 2,3,5. Solution: When a die is thrown once, total possible outcomes =6 and prime numbers in it are (2,3,5). Total possible outcomes = 3 Probability of getting a prime = $\frac{3}{6}=\frac{1}{2}$. Ans-(d)	1

	SEC-B	
6	Two different dices are tossed together. Find the probability (a) of getting a Doublet (b) of getting a sum 10,of the numbers on the two dice. **Solution**: When two dice are tossed together, Total possible outcomes= 6X6=36 i) The favourable outcomes = (1,1),(2,2) ,(3,3),(4,4),(5,5),(6,6) No. of favourable outcomes=6 Probability of getting doublet= 6/36=1/6 ii) No. of favourable outcomes=(4,6),(5,5),(6,4) No. of favourable outcomes=3 Probability of getting a sum 10= 3/36=1/12	2
7	Two dice are thrown at the same time. Find the probability that the sum of the two numbers appearing on the top of the dice is more then 9. Solution: Total possible outcomes = n(s) = 6 × 6 = 36. Let event E be the event that sum of two numbers appearing on the top is more than 9. Hence, Favourable outcomes of event E = {(4, 6), (5, 5), (5, 6), (6, 5), (6, 6), (6, 4)} Hence, Total outcomes favourable to event E is n(E) = 6. ∴ P(E) = 6/36=1/6	2
8	A card is drawn at random from a well shuffled pack of 52 cards. Find the probability that the drawn card is neither a king nor a queen. **Solution:**Total possible outcomes= 52 Let E be the event that the drawn card is neither a king nor a queen. Total no. of kings and queens=4+4=8 Therefore 52-8=44 cards that are neither king or queen. No. of favourable outcomes=44 Probability (E)= 44/52=11/13	2
	SEC-C	

9	A game consists of tossing a one-rupee coin 3 times and noting the outcomes each time. Sachin wins the game if all the tosses give the same result and loses otherwise. Find the probability of Sachin losing the game. **Solution:**We use the basic formula of favourable outcomes to solve the problem. Total possible outcomes are ={HHH, TTT, HTH, HHT, THH, THT, TTH, HTT} = 8 Number of possible outcomes to get three heads or three tails is 2 The probability that Sachin will win the game = Number of possible outcomes/Total number of favourable outcomes= 2/8= 1/4 The probability that Sachin will lose the game is 1 - ¼ = 3/4 The probability that Sachin will lose the game is 3/4.	3
10	A box contains 12 balls of which some are red in colour. If 6 more red balls are put in the box and a ball is drawn at random, the probability of drawing a red ball doubles than what it was before. Find the number of red balls in the bag. Answer: Let there be 'x' no. of red balls in the box originally ∴ P(red ball) = x/12 After adding 6 red balls P(red ball) = x+6/12+6 =x+6/18 .It is given that $\frac{x+6}{18} = 2\left(\frac{x}{12_6}\right)$ ⇒ x + 6 = 3x ⇒ 2x = 6 ⇒ x = 3 ∴ There are 3 red balls in the bag.	3
11	A bag contains numbers which are numbered from 2to 90. A card is drawn at random from the bag. Find the probability that it bears i) a two-digit number ii) a number which is a perfect square. **Solution:**Let Ebe the event of drawing a two digit number from the cards	3

	numbered from 2 to 90 are 10,11,12,…….90. No of favourable outcomes=81 Total no of possible outcomes=89 P(E)= 81/89 iii) Let F be the event of drawing aa perfect square number from the cards numbered from 2 to 90 are 4,9,16,25,36,49,64,81, No of favourable outcomes=8 Total no of possible outcomes=89 P(F)= 8/89	
	CASE BASED QUESTIONS	
12	Two friend Dhoni and sachin are travelling in a train. They feeling bored, so they started played a game with a pair of dice that one of them had, each of them started rolling the pair of dice one by one, starting one condition before rolling. If the person gets the numbers according to the condition stated by him, he wins and get a score. Answer the following questions: i) Dhoni says, "a doublet". What is the probability of his winning? **or** ii) Sachin says, : sum less than 9:. What is the probability of his winning? iii) Dhoni says, " 6 will come up either time ". What is the probability of his winning? iv) Sachin says ," sum is an even number ". What is the probability of his losing? **Solution**:Let's analyse each friend's condition: i) The probability of rolling a doublet with a pair of dice is 1/6, as there are six possible doublets (1,1),(2,2), (3,3), (4,4), (5,5), (6,6). So, the probability of the first friend winning is 6/36=1/6. **OR** ii)The possible outcomes with a sum less than 9 (1,1), (1,2), (1,3), (1,4), (1,5),(1,6) (2,1), (2,2), (2,3), (2,4),(2,5),(2,6) (3,1), (3,2), (3,3), (3,4),(3,5),(4,1), (4,2), (4,3),(4,4),(5,1), (5,2),(5,3)(6,1),(6,2)	2 2 1 1

	which makes a total of 26 favourable outcomes. There are a total of 6 X6 = 36 possible outcomes with a pair of dice. So, the probability of the second friend winning is 26/36, =13/18 iii)Possible outcomes when 6 will come up time are (1,6), (2,6), (3,6), (4,6), (5,6),(6,1) (6,2), (6,3), (6,4),(6,5),(6,6) No of favourable outcomes =11 So, the probability of the first friend winning is 11/36. d. The possible outcomes with an even sum are (1,1), (1,3), (1,5), (2,2), (2,4), (2,6), (3,1), (3,3), (3,5), (4,2), (4,4), (4,6), (5,1), (5,3), (5,5), (6,2), (6,4), (6,6), making a total of 18 favourable outcomes. There are 6 X6 = 36 possible outcomes, so the probability of the second friend losing is 18/36=1/2 Probability of his losing is 1-1/2=1/2	
13	Raju , Rishi and Rakesh are three friends .They are playing card games. A card is drawn at random from a well shuffled pack of 52 playing cards. A. Find the probability of getting neither a red card nor a queen. B. What is the probability of getting "a black¬face card' from a well shuffled deck of 52 playing cards?  Solution A. Let E be the event of getting neither a red card nor a queen. No. of red cards = 26 (including 2 queens) Remaining (black) queens = 2	1 2 1

	Neither red nor queen = 52 – (26 + 2) = 52 – 28 = 24 P(E) = 24/52=6/13 B. There are 6 black face cards. (2 red kings, 2 red queens and 2 jacks) Total cards = 52 ∴ Required Probability = 6/52=3/26	
	PRATICE SET (UNSOLVED)	
	MCQ	
1	A bag contains three green marbles, four blue marbles and two orange marbles .If a marble is picked at random , then the probability that it is not an orange marble is (a) 3/9 (b) 7/9 (c) 4/9 (d) none of these	1
2	A card is drawn from a deck of 52 cards . The event E is that card is not an ace of hearts. The numbers of outcomes favourable to E is (a)13 (b) 51 (c) 48 (d) none of these	1
3	A die is thrown once, the probability of getting an odd numbers less then 3 is (a) 1/6 (b) 1/2 (c) 1/3 (d) none of these	1
4	The probability that a number selected at random from the numbers 1,2,3…..15 is a multiple of 4 is (b) 1/5 (b) 2/15 (c) 1/3 (d) none of these	1
5	The question consist of two statement – Assertion (A) and Reason. Answer the question selecting the appropriate option given below: (a)Both A and R are true is the correct exploration for A. (b)Both A and R are true and R is not the correct exploration for A. (c)A is true but R is false. (d)A is false but R is true. Assertion (A) : The probability of winning a game is 0.5 , then the	1

	probability or loosing it, is 0.5 Reason (R) : P(E) +P(not E) =1.	
	SEC-B	
6	It is given that in a group of 5 students, the probability of 4 students not having the same birthday is 0.892. What is the probability that the 2 students have the same birthday?	2
7	100 tickets of a lottery were sold and there are 5 prizes on these tickets. If Saket has purchased one lottery ticket, what is the probability of winning a prize?	2
8	Find the probability that a non-leap year selected at random will contain 53 Sundays.	2
	SEC-C	
9	One card is drawn from a well – shuffled deck of 52 cards. Find the probability of getting: i) A face card ii) A spade iii) the jack of diamonds	3
10	Cards numbered from 11-60 are kept in a box. If a card is drawn at random from the box, find the probability that the number on the drawn card is: i) A perfect square number ii) divisible by 5	3
11	A bag contains 10 red, 5 blue and 7 green balls. A ball is drawn at random. Find the probability of this ball being not a blue ball. (Ans: 17/22)	3
	CASE BASED QUESTIONS	
12	Rajdeep and Sumit planned to play board game in which they were supposed to use two dice.  i)Rajdeep got first chance to roll the dice.What is the the probability that he got	2 1 1

	the sum of the two numbers appearing on the top face of the dice as 8 ? **OR** ii)Sumit got next chance. What is the probability that he got the sum of the numbers appearing on the top of the dice as 13 ? i) Rajdeep got next chance to roll the dice.What is the the probability that he got the sum of the two numbers appearing on the top face of the dice less than or equal to 12 ? ii) Sumit got first chance to roll the dice.What is the the probability that he got the sum of the two numbers appearing on the top face of the dice as 7 ?	
13.	Ashmita is go to visit Rath mela . She want to play a game spinning an arrow which comes to rest pointing at one of the numbers 1,2,3,4,5,6,7,8 .These are likely outcomes.  i) What is the probability that it will point at 8? **OR** ii) What is the probability that it will point at odd number? iii) What is the probability that it will point at a number greater than 2? iv) What is the probability that it will point at a number less than 9?	2 1 1

PRACTICE SET ANSWERS

1.Ans-(b) 7/9

2.Ans-(b) 51

3.Ans-(a) 1/6

4.Ans -(a) 1/5

5.ANS- (a) Both A and R are true is the correct exploration for A.

6.Ans: 0.108

7.Ans: 1/20

8.Ans:1/7

9.Ans: i)3/13 ii) ¼ iii) 1/52

10.Ans: i)2/25 ii) 1/5

11.Ans:17/22

CASE BASED

12 i) Ans:5/36ii) Ans:0/36=0iii) Ans:36/36=1iv) Ans:6/36=1/6

13.Ans: i)1/8ii)4/8=1/2iii)6/8=3/4iv)8/8=1

Sample Question Paper -1
Class X
Basic Mathematics (241)

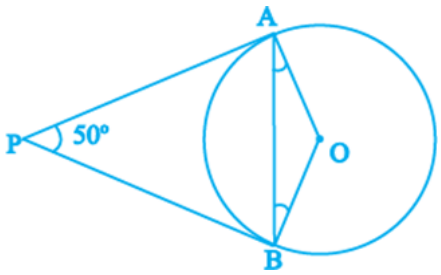
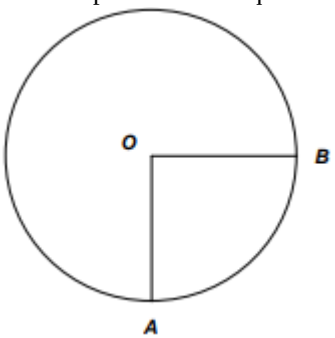
Time Allowed: 3 Hrs **Maximum Marks: 80**

General Instructions:

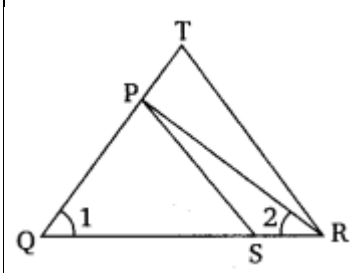
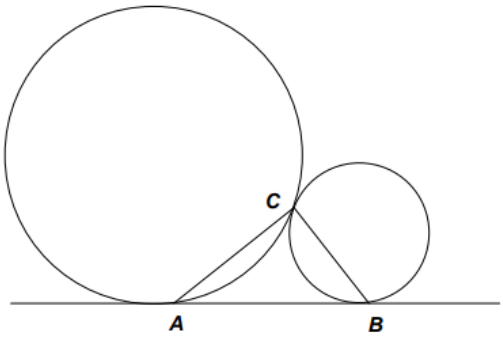
1. **This Question Paper has 5 Sections A, B, C, D, and E.**
2. Section A has 20 Multiple Choice Questions (MCQs) carrying 1 mark each.
3. **Section B has 5 Short Answer-I (SA-I) type questions carrying 2 marks each.**
4. Section C has 6 Short Answer-II (SA-II) type questions carrying 3 marks each.
5. **Section D has 4 Long Answer (LA) type questions carrying 5 marks each.**
6. Section E has 3 sourced based/Case Based/passage based/integrated units of assessment (4 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.
7. **All Questions are compulsory. However, an internal choice in 2 Qs of 2 marks, 2 Qs of 3 marks and 2 Questions of 5 marks has been provided. An internal choice has been provided in the 2 marks questions of Section E.**
8. **Draw neat figures wherever required. Take $\pi = 22/7$ wherever required if not stated.**

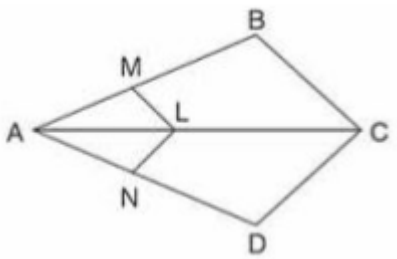
Section A

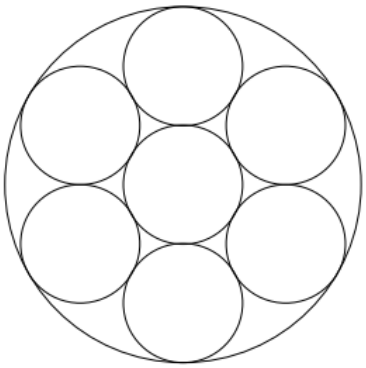
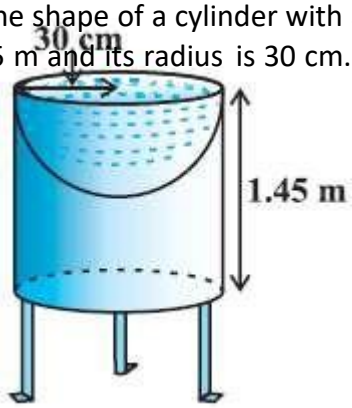
	Section A consists of 20 questions of 1 mark each.	
SN		**Marks**
1	If n is any natural number, then $6^n - 5^n$ always ends with (a)1 (b) 3 (c) 5 (d) 0	1
2	If two positive integers m and n are expressible in the form $m = pq^3$ and $n = p^3q^2$; p,q being prime numbers , then HCF(m,n) is (a)pq (b) pq^2 (c)) p^3q^3 (d)) p^2q^3	1
3	A quadratic polynomial, the sum of whose zeroes is 0 and one zero is 3, is (a) x^2+3 (b) x^2-3 (c) x^2+9 (d) x^2-9	1
4	If the system of equations $3x - y+8 = 0$, $6x - ky+16 = 0$ represent coincident lines , then k = (a) -1/2 (b) 1/2 (c) 2 (d) -2	1
5	If the quadratic equation $x^2 + 4x + k = 0$ has real and equal roots, then (a) $k < 4$ (b) $k > 4$ (c) $k = 4$ (d) $k \geq 4$	1

6	If the distance between the points (4,p) and (1,0) is 5 , then the value of p is (a)4 only (b) ± 4 (c) -4 only (d) 0	1
7	If ΔPQR ~ ΔXYZ and XY = 4cm , YZ = 4.5 cm, ZX = 6.5 cm and PQ = 8cm, then perimeter of ΔPQR is (a) 25 cm (b) 23 cm (c) 15 cm (d) 30 cm	1
8	In ΔABC and ΔDEF , $\angle B = \angle E, \angle F = \angle C$ and AB = 3DE . Then the two triangles are (a) Congruent but not similar (b) Similar but not congruent (c) neither congruent nor similar (d) congruent as well as similar	1
9	 In the figure, if PA and PB are tangents to the circle with centre O such that ∠APB = 50°, then ∠OAB is equal to (a) 25° (b) 30° (c) 40° (d) 50°	1
10	If tanA = 4/3 , then the value of secA is (a) 4/5 (b) 3/5 (c) 3/4 (d) 5/3	1
11	If cosA =3/5 , then the value of 9 + 9 $\tan^2$A is equal to (a) 9 (b) 1 6 (c) 25 (d) 34	1
12	The value of $\tan^2 45^\circ - \sin^2 45^\circ$ (a) 1/4 (b) 3/4 (c) 1/2 (d) 0	1
13	What is the perimeter of a quadrant of a circle (OAB) whose diameter is 10cm? (Use π = 3.14)  (a) 7.85cm (b) 17.85cm (c) 27.85cm (d) 37.85cm	1

14	The area of the circle that can be inscribed in a square of side 6 cm is (a) 36 π cm^2 (b) 18 π cm^2 (c) 12 π cm^2 (d) 9 π cm^2	1
15	If a solid sphere with total surface area 48cm^2 is bisected into two hemispheres, then the total surface area of any one of the hemisphere is (a) 48 cm^2 (b) 60 cm^2 (c) 24 cm^2 (d) 36 cm^2	1
16	For the following distribution : Class: 0 - 5 | 5 - 10 | 10 - 15 | 15 - 20 | 20 - 25 Frequency: 10 | 15 | 12 | 20 | 9 the upper limit of the median class is (a) 10 (b) 15 (c) 20 (d) 25	1
17	If the sum of the 15 observations of a data is (434+x) and the mean of the observation is x, , then the value of x is (a) 25 (b) 27 (c) 31 (d) 33	1
18	A card is selected at random from a well shuffled deck of 52 cards. The event E is that card is not an ace of hearts. The number of outcomes favourable to E is (a) 4 (b) 12 (c) 48 (d) 51	1
	Direction for questions 19 & 20: In question numbers 19 and 20, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option.	
19	**Assertion (A):** If product of two numbers is 12960 and their HCF is 18, then their LCM is 720. **Reason (R):** HCF is always a factor of LCM (a)Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.	1
20	**Assertion (A):** The ratio in which the line segment joining A (5, 3) and B (-3, 11) internally divided by the point C(3,5) is 1:3. **Reason (R):** as formula for the internal division is $\left(\frac{mx2 + nx1}{m+n}, \frac{my2 + ny1}{m+n}\right)$ (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true but Reason (R) is false.	1

	(d) Assertion (A) is false but Reason (R) is true.	
	Section B	
	Section B consists of 5 questions of 2 marks each.	
21	For which values of p will the following pair of linear equations given below has unique solution? $4x + py + 8 = 0$ $2x + 2y + 2 = 0$	2
22	ABCD is a trapezium in which AB ‖ DC and its diagonals intersect each other at the point O. Show that $\frac{OA}{OC} = \frac{OB}{OD}$ OR, **In the given figure, $\frac{QR}{QS} = \frac{QT}{PR}$ and ∠1 = ∠2. show that ΔPQR ~ ΔTQR.** 	2
23	In figure 1 above, two circles touch each other externally at C, and AB is a common tangent of circles, then find ∠ACB. 	2
24	Evaluate: $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$	2
25	Find the area of a quadrant of a circle whose circumference is 44 cm. **[OR]** Find the diameter of a circle whose area is equal to the sum of the areas of the two circles of radii 24 cm and 7 cm.	2

	Section C	
	Section C consists of 6 questions of 3 marks each.	
26	Prove that $\sqrt{2}$ is an irrational number.	3
27	Form a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $5x^2 + 2x - 3$.	3
28	Ram and Sham are two friends in a town; both have their own plots. Ram is an owner of a rectangular plot whose perimeter is 50m and Sham is also the owner of a rectangular plot whose perimeter is 100m. Sham's plot has a length twice that of Ram's plot and breadth is 5m more than that of Ram's plot. Find the dimensions of Ram's and Sham's plot. **[OR]** Two numbers, x and y (x > y), have a difference of 6 and an average of 4. Frame a pair of linear equations in two variables. Determine the values of the two numbers.	3
29	Prove that the parallelogram circumscribing a circle is a rhombus.	3
30	**Prove that** $\frac{sin\theta - 2sin^3\theta}{2cos^3\theta - cos\theta} = tan\theta$ **[OR]** If sin θ + cos θ = $\sqrt{3}$, then prove that tan θ + cot θ = 1	3
31	Apoorva throws two dice once and computes the product of the numbers appearing on the dice. Peehu throws one die and squares the number that appears on it. Who has the better chance of getting the number 36? Why?	3
	Section D	
	Section D consists of 4 questions of 5 marks each.	
32	An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains. **[OR]** A motor boat whose speed is 18 km/h in still water takes 1 hour more to go 24 km upstream than to return downstream to the same spot. Find the speed of the stream.	5
33	Prove that If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. In given figure, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$. 	5

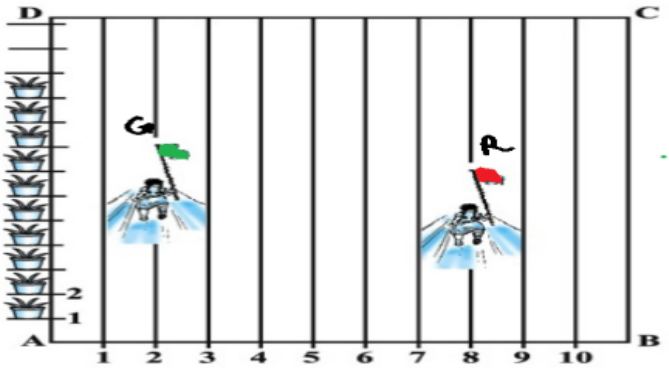
34	Kanupriya runs a bakery shop. The amount of mixture required to make one biscuit is 18 $cm3$. After the biscuit is cooked, it becomes a cylinder of radius 3 cm and height 0.7 cm and has some air trapped inside it. Biscuits are packed in a cylindrical card box of height 14 cm. The arrangement of biscuits is shown above. Based on this information, answer the following questions: (i)How many biscuits will be there in a box? (ii)Find the volume of one biscuit after it is cooked. (iii)Find the volume of air trapped in the biscuit.  **[OR]** Ramesh made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end. The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath. 	5
35	The following table shows the ages of the patients admitted in a hospital during a month. Find the mode and the mean of the data given below: Age (in years) | Number of patients 5-15 | 6 15-25 | 11 25-35 | 21 35-45 | 23 45-55 | 14 55-65 | 5	5
	Section E	
	Case study based questions are compulsory.	

36 **Case Study – 1**

Your elder brother wants to buy a car and plans to take loan from a bank for his car. He repays his total loan of Rs 1,18,000 by paying every month starting with the first instalment of Rs 1000. If he increases the instalment by Rs 100 every month . Based on the above information answer the following questions.

I.	Find the amount paid by him in 30th installment.	1
II.	If total installments are 40 then find the amount paid in the last installment?	1
III.	Find the amount paid by him in the 30 installments. **[OR]** What amount does he still have to pay after 30th installment?	2

37 In order to conduct Sports Day activities in your School, lines have been drawn with chalk powder at a distance of 1 m each, in a rectangular shaped ground ABCD, 100 flowerpots have been placed at a distance of 1 m from each other along AD, as shown in given figure below. Niharika runs 1/4 th the distance AD on the 2nd line and posts a green flag. Preet runs 1/5 th distance AD on the eighth line and posts a red flag.



I.	Find the position of green flag.	1
II.	What is the distance between green flag and red flag?	1

		III.	If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag? **[OR]** If Joy has to post a flag at one-fourth distance from green flag, in the line segment joining the green and red flags, then where should he post his flag?	2

38 **Case Study – 3**

A group of students of class X visited India Gate on an education trip. The teacher and students had interest in history as well. The teacher narrated that India Gate, official name Delhi Memorial, originally called All-India War Memorial, monumental sandstone arch in New Delhi, dedicated to the troops of British India who died in wars fought between 1914 and 1919.The teacher also said that India Gate, which is located at the eastern end of the Rajpath (formerly called the Kingsway), is about 138 feet (42 metres) in height.

I.	What is the angle of elevation if they are standing at a distance of 42m away from the monument?	1
II.	They want to see the tower at an angle of 60° . So, they want to know the distance where they should stand and hence find the distance.	1
III.	If the altitude of the Sun is at 60 °, then find the height of the vertical tower that will cast a shadow of length 20 m. **[OR]** The ratio of the length of a rod and its shadow is 1:1. Find the angle of elevation of the Sun.	2

MARKING SCHEME
SAMPLE PAPER-1

	Section A	
1	(a) 1 For any natural number 6^n and 5^n end with 6 and 5 respectively. Hence $6^n - 5^n$ always end with 6-5 =1 .	1
2	(b) pq^2	1
3	$\alpha + \beta = 0 \Longrightarrow 3+\beta = 0 \implies \beta = -3. Hence\ polynomial = (x+\alpha)(x+\beta) = (x+3)(x-3)$ (d) x^2-9	1
4	(b) 2 $a_1/a_2 = b_1/b_2 = c_1/c_2 \Longrightarrow 3/6 = 1/k = 8/16$ Hence k =2	1
5	(c) k = 4 $b^2 - 4ac = 0 \Longrightarrow 16 - 4k = 0 \Longrightarrow k = 4$	1
6	(b) ± 4 $(4-1)^2 + (p-0)^2 = 5^2 \Longrightarrow 9 + p^2 = 25 \Longrightarrow p^2 = 16$. Hence $p = \pm 4$	1
7	(d) 30 cm	1
8	(b) Similar but not congruent	1
9	(a) 25° $\{\angle APB + \angle AOB = 180^\circ \Longrightarrow \angle AOB = 180^\circ - 50^\circ = 130^\circ$ In ΔAOB, $\angle OBA = \angle OAB$(angles opposite to equal sides); $\angle OAB = (180^\circ - 130^\circ)/2$	1
10	(d) 5/3	1
11	(b) 25 $9(1 + \tan^2 A) = 9\sec^2 A = 9/\cos^2 A = 9 \times 25/9 = 25$	1
12	(c) ½ $\tan^2 45^\circ - \sin^2 45^\circ = 1 - 1/2 = 1/2$	1
13	(b) 17.85cm perimeter of a quadrant of a circle (OAB)= $r+r+l = 2r + 2\frac{\pi r}{4} = d + \frac{\pi d}{4} = 10 + \frac{3.14 \times 10}{4} = 17.85$	1

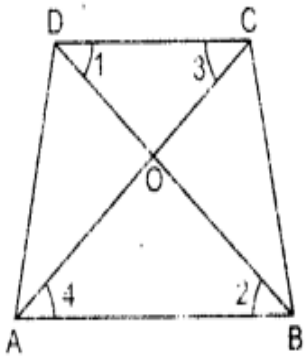
14	(d) 9 π cm^2 d = 6 , r =3, A = $\pi r^2 = 9\pi$	1
15	(d) 36 cm^2 $4\pi r^2 = 48 \Longrightarrow 3\pi r^2 = 36$	1
16	(b) 15 N= 10+15+12+20+ 9 = 66; N/2= 33 ; 33rd frequency lies in class interval 10-15 from beginning. Median class = 10-15. Upper limit of median class = 15	1
17	(d) 31 $(434+x)/15 = x \Longrightarrow 15x = 434+x \Longrightarrow 14x = 434 \Longrightarrow x = 31$	1
18	(d) 51	1
19	(b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).	1
20	(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).	
	SECTION B	
21	For unique solution : $a_1/a_2 \neq b_1/b_2$ $\Longrightarrow 4/2 \neq p/2$ $\Longrightarrow p \neq 4$ Hence, for all values of p , except 4 , the given pair of linear equations will have a unique solution.	½ ½ ½ ½

22

Given: Diagonals AC and BD intersect at O. ½

$AB \parallel DC$

To Prove: $\frac{OA}{OC} = \frac{OB}{OD}$



Proof: In ΔAOB and ΔCOD

$\angle 1 = \angle 2$ ½

$\angle 3 = \angle 4$ [Alternate angles] ½

$\therefore \quad \Delta AOB \sim \Delta COD$ [AA] ½

$\Rightarrow \quad \frac{OA}{OC} = \frac{OB}{OD}$ [Corresponding sides of similar triangles] ½

Or, ½

From the figure,

$\angle 1 = \angle 2$

$\therefore PQ = PR$ [Sides opposite to equal angles are equal]

In ΔPQS and ΔTQR ½

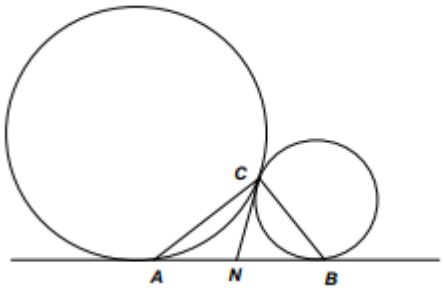
$\Rightarrow \frac{QR}{QS} = \frac{QT}{PR}$

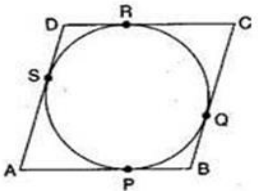
$\Rightarrow \frac{QR}{QS} = \frac{QT}{PR}$ [$\because$ PQ = PR proved above] ½

$\angle PQS = \angle TQR = \angle 1$

$\therefore \Delta PQS \sim \Delta TQR$ [By SAS similarity] ½

Hence, **proved.**

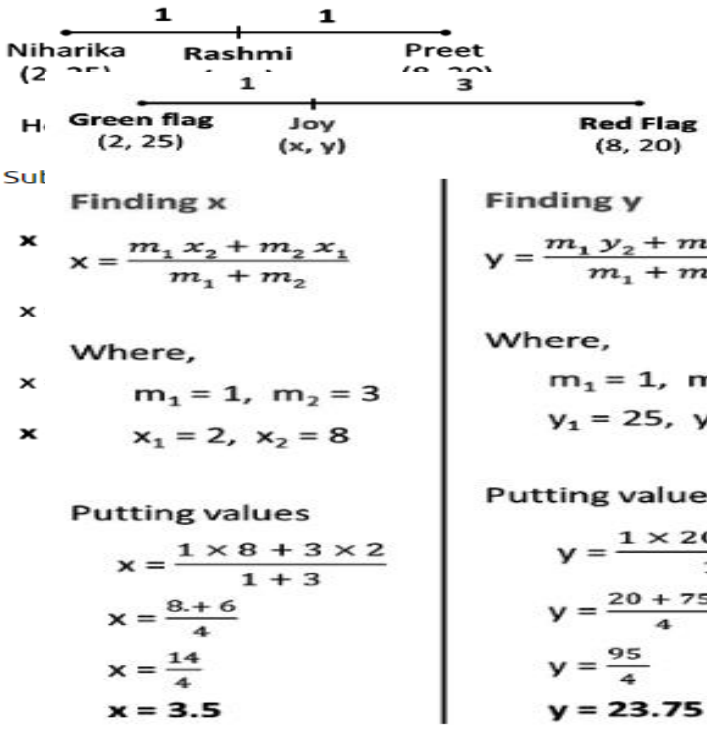
23	 We also know that angle opposite to equal sides is equal. Therefore ∠NCA = ∠NAC and ∠NCB = ∠NBC ∠NCA + ∠NCB = ∠NAC + ∠NBC ∠NCA + ∠NCB + ∠NAC + ∠NBC = 180° ∠NCA + ∠NCB = 90°	½ ½ ½ ½
24	$2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$ $= 2\times1 + \frac{3}{4} - \frac{3}{4}$ $= 2$	1 1
25	R= 44cm/2π =7cm Area of Quadrant = $\pi r^2/4$ $= 22\times49\text{cm}^2/4 = 269.5\ \text{cm}^2$ Or, Area of the circle = Area of first circle + Area of second circle $\Rightarrow \pi R^2 = \pi (r_1)^2 + \pi (r_1)^2$ $\Rightarrow \pi R^2 = \pi (24)^2 + \pi (7)^2 \Rightarrow \pi R^2 = 576\pi + 49\pi$ $\Rightarrow \pi R^2 = 625\pi \Rightarrow R^2 = 625 \Rightarrow R = 25$ Thus, diameter of the circle = 2R = 50 cm	½ ½ 1 ½ ½ 1
	SECTION C	
26	Let us assume to the contrary, that $\sqrt{2}$ is rational. Then we can find a and b ($\neq 0$) such that $\sqrt{2}$ =a/b (assuming that a and b are co-primes). So, $a = \sqrt{2}\, b \Rightarrow a^2 = 2b^2$ Here 2 is a prime number that divides a^2 then 2 divides a also (Using the theorem, if a is a prime number and if a divides p^2, then a divides p, where a is a positive integer) Thus 2 is a factor of a	1 ½

	Since 2 is a factor of a, we can write a = 2c (where c is a constant). Substituting a = 5c We get $(2c)^2 = 2b^2 \Rightarrow 2c^2 = b^2$ This means 2 divides b^2 so 2 divides b also (Using the theorem, if a is a prime number and if a divides p^2, then a divides p, where a is a positive integer). Hence a and b have at least 2 as a common factor. But this contradicts the fact that a and b are coprime. This is the contradiction to our assumption that p and q are co-primes. So, √2 is not a rational number. Therefore, the √2 is irrational.	½ ½ ½
27	$\alpha+\beta=-\frac{2}{5}\ldots(1)$ $\alpha\beta=\frac{-3}{5}\ldots(2)$ $\frac{1}{\alpha}+\frac{1}{\beta}=\frac{\alpha+\beta}{\alpha\beta}$ $=\frac{\frac{-2}{5}}{\frac{-3}{5}}(\text{from}(1))$ $=\frac{2}{3}$ $\frac{1}{\alpha}\times\frac{1}{\beta}=\frac{1}{\alpha\beta}$ $=\frac{-5}{3}(\text{ from }(2))$ -b/a =2/3 and c/a = -5/3 a=3, b=-2, c =-5 Required polynomial = $ax^2+ bx +c = 3x^2 -2x -5$	1 ½ ½ ½ ½
28	Let length and Breadth of Ram's plot be x m and y m respectively. x+y=25, 2x+(y+5) =50 x= 20, y=5 Length =20m, breadth= 5m of Ram's plot Sham's plot-Length =40m, breadth= 10m Or, x-y = 6 (x+y)/2 =4 X=7 , y=1	1 1 1 ½ ½ 2
29	 Since, the tangents from an external point to a circle are equal. AP = AS …….(i) BP = BQ …….(ii) CR = CQ …….(iii)	1

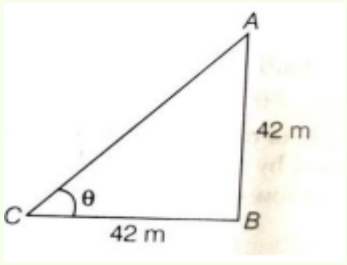
	DR = DS …….(iv) On adding eq. (i), (ii), (iii) and (iv), we get (AP + BP) + (CR + DR) = (AS + BQ) + (CQ + DS) AB + CD = (AS + DS) + (BQ + CQ) AB + CD = AD + BC AB + AB = AD + AD [Opposite sides of gm are equal] 2AB = 2AD AB = AD But AB = CD and AD = BC [Opposite sides of gm] AB = BC = CD = AD Parallelogram ABCD is a rhombus.	1 1
30	$$\frac{\boldsymbol{sin}\theta - \boldsymbol{2sin^3}\theta}{\boldsymbol{2cos^3}\theta - \boldsymbol{cos}\theta}$$ $$= \frac{\boldsymbol{sin}\theta(\boldsymbol{1 - 2sin^2}\theta)}{\boldsymbol{cos}\theta(\boldsymbol{2cos^2}\theta - \boldsymbol{1})}$$ $$= \frac{\boldsymbol{tan}\theta(\boldsymbol{1 - 2sin^2}\theta)}{\boldsymbol{2(1 - sin^2}\theta) - \boldsymbol{1}}$$ $$= \frac{\boldsymbol{tan}\theta(\boldsymbol{1 - 2sin^2}\theta)}{(\boldsymbol{1 - 2sin^2}\theta)}$$ **= tan**θ	1 1 1
	[OR] $\sin\theta + \cos\theta = \sqrt{3} \Rightarrow (\sin\theta + \cos\theta)^2 = 3$ $\Rightarrow \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$ $\Rightarrow 1 + 2\sin\theta\cos\theta = 3 \Rightarrow \sin\theta\cos\theta = 1$ Now $\tan\theta + \cot\theta = {}^{\sin\theta}/_{\cos\theta} + {}^{\cos\theta}/_{\sin\theta}$ $= (\sin^2\theta + \cos^2\theta)/\sin\theta\cos\theta$ $= 1/\sin\theta\cos\theta$ $= 1/1 = 1$	½ ½ ½ ½ ½ ½
31	Apoorva throw two dice at once. Number of outcomes for getting product 36 = 1(6×6) ∴ Probability for Apoorv = 1/36 Peehu throws one die Hence, the total number of outcomes = 6 Number of outcomes for getting square36 = 1 ∴ Probability for Peehu = 6/36 = 1/6 Therefore, Peehu has a better chance of getting the number 36.	1 1 1

	Section D	
32	Let the average speed of the passenger train be x km/hr. Then the average speed of express train = (x+11)km/hr $\frac{132}{x} - \frac{132}{x+11} = 1$ $\Rightarrow x^2+11x-1452=0$ $\Rightarrow (x-33)(x+44)=0$ $\Rightarrow (x-33)=0$ or $(x+44)=0$ $\Rightarrow x=33$ or $x=-44$ Speed can not be negative, $\Rightarrow x=33$, $x+11=44$ Hence, the speed of passenger train =33 km/hr and the speed of express train =44 km/hr [OR] Let x be the speed of stream. Let t1 and t2 be the time for upstream and downstream. Now , t1 = t2 +1 $\Rightarrow 48x=(18-x)(18+x)$ $\Rightarrow 48x=324+18x-18x-x^2$ $\Rightarrow x^2+48x-324=0$ $\Rightarrow x^2+54x-6x-324=0$ $\Rightarrow x(x+54)-6(x+54)=0$ $\Rightarrow (x+54)(x-6)=0$ $\Rightarrow x=-54$ or $x=6$ Since speed cannot be negative. $\therefore x=6$ Thus the speed of stream is 6km/hr	1/2 1½ 1 1 1
33	Figure Given, To prove, constructionsProof Application ----	½ 1 ½ 2 1
34	In a layer, 7 biscuits are arranged whose height is 0.7 cm. Total layer in box = 14/0.7 = 20 Number of biscuits in the box = 20 x 7 = 140 19.8 cm^3 (Volume of cylinder = $\pi r^2 h$)	½ 1 2 1½

	Volume of air trap= Volume of biscuit−Volume of sphere = 19.8 −18 = 1.8 cm^3 **[OR]** Let h be height of the cylinder, and r the common radius of the cylinder and hemisphere. Then, the total surface area = CSA of cylinder + CSA of hemisphere $= 2\pi rh + 2\pi r^2 = 2\pi r (h + r)$ $= 2 \times \frac{22}{7} \times 30 (145 + 30)\ cm^2$ $= 2 \times \frac{22}{7} \times 30 \times 175\ cm^2$ $= 33000\ cm^2 = 3.3\ m^2$	1 1 1 1 1
35	Mode: The class which have highest frequency. In this case, class interval 35−45 is the modal class. Now, Lower limit of modal class, l=35, h=10, f1=23, f0=21, f2=14 We know that, $\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$ $= 35 + \left(\frac{23 - 21}{2(23) - 21 - 14}\right) \times 10$ $= 35 + \frac{2}{11} \times 10$ $= 35 + 1.818$ Mode =36.8 Lets take assumed mean, A as 30 Class height =10 $\therefore u_i = \frac{x_i - A}{h} = \frac{x_i - 30}{10}$ $\bar{x} = A + h\frac{\sum f_i u_i}{\sum f_i} = 30 + 10 \times \frac{43}{80}$ =30+5.375 =35.375 ≈35.37 Hence, Mode =36.8 years, Mean =35.37 years.	½ **1** **1** **1** **1** **1/2**
	SECTION E	

36	amount paid by him in 30th installment = a+29d = 1000+29×100= 3900 amount paid in the last installment = a+39d = 1000+39×100= 4900 amount paid by him in the 30 installments= n/2 [2a+ (n-1)d] = 30/2 [2×1000 +(30-1)100] = 73500 OR, amount he still have to pay after 30th installment= 118000- 73500 = 44500	**1** **1** **2**
37	Position of the green flag is (2, ¼ ×100) i.e., (2,25) the distance between green flag and red flag = $= \sqrt{(8-2)^2 + (20-25)^2}$ $= \sqrt{(6)^2 + (-5)^2}$ $= \sqrt{(6)^2 + (5)^2}$ $= \sqrt{36+25}$ $= \sqrt{61}$ (i) Position of the blue flag = (5,22.5)  Or, Position of Joy flag is (3.5, 23.75) (i) To find the angle of elevation, tanθ = height of the monument/ distance ofstudents from the monument = 42 /42	**1** **1** **2**

38



=1= tan 45°

Hence, θ =45°

(ii)

To find the distance, $\tan 60° = \frac{\text{height of the tower}}{\text{distance}}$

$\sqrt{3} = \frac{42}{\text{distance}}$

$\text{distance} = \frac{42}{\sqrt{3}}$

distance = 24.64 m

1

(iii)

To find the height of the vertical tower, $\tan 60° = \frac{\text{height of the tower}}{\text{distance}}$

$\sqrt{3} = \frac{\text{height of the tower}}{20}$

Height of the tower = $20\sqrt{3}$

1

(iv)

To find the angle of elevation of the sun, $\tan \theta = \frac{\text{height of the tower}}{\text{distance from the tower}}$

$= \frac{1}{1}$ (since the ratios are in 1:1)

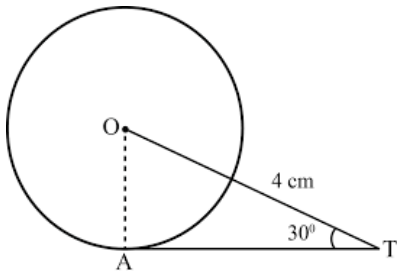
θ =45°

2

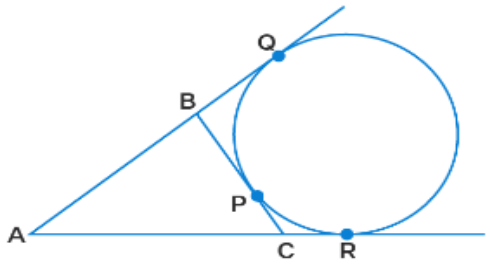
Sample Question Paper-2

Class:X **Mathematics Standard(041)**

Time Allowed: 3 Hrs **Maximum Marks: 80**

1	In the given figure, AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30°, then AT = (a) 4 cm (b) 2 cm (c) 2√3 cm (d) 4√3 cm  	1
2	If the coordinates of one end of a diameter of a circle are (2, 3) and the coordinates of its centre are (-2, 5), then the coordinates of the other end of the diameter are (a) (0, 8) (b) (0, 4) (c) (6, -7) (d)(-6, 7)	1
3	AOBC is a rectangle whose three vertices are vertices A (0, 3), O (0, 0) and B (5, 0). The length of its diagonal is (a) 5 (b) 3 (c) √34 (d) 4	1
4	A bag has 5 white marbles, 8 red marbles and 4 purple marbles. If we take a marble randomly, then what is the probability of not getting purple marble? (a) 0.5 (b) 0.66 (c) 0.08 (d) 0.77	1
5	In what ratio does the x-axis divide the join of A(2, -3) and B(5, 6)? (a) 1 : 2 (b) 3 : 5 (c) 2 : 1 (d) 2 : 3	1
6	If the lines 3x - 2ky – 2 = 0 and 2x + $\frac{5}{3}$ y + 1 = 0 are parallel, then what is the value of k? (a) 4/15 (b) - 5/4 (c) 4/5 (d) - 15/4	1
7	If k+1, 3k and 4k+2 be any three consecutive terms of an AP, find the value of k. (a) 3 (b) -3 (c) 4 (d) 1	1
8	The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is: (a) 10 (b) 100 (c) 504 (d) 2520	1
9	The relation between Mean, Mode and Median is (a) Mode = (3 x Mean) - (2 x Median) (b) Mode = (2 x Median) - (3 x Mean) (c) Mean = $\frac{1}{2}$ (Mode – 3 x Median) (d) Median = (3 x Mean) - (2 x Mode)	1
10	A sphere of diameter 18 cm is dropped into a cylindrical vessel of diameter 36 cm, partly filled with water. If the sphere is completely submerged, then the water level rises by (a) 4cm (b) 5 cm (c) 3 cm (d) 6 cm	1
11	One card is drawn at random from a well-shuffled deck of 52 cards. What is the probability of getting a black face card? (a) 3/26 (b) 3/14 (c) 3/26 (d) 1/26	1

12	The value of $\sqrt{6+\sqrt{6+\sqrt{6+}}}$ ….. is: (a) 4 (b) 3 (c) 3.5 (d) -3	1
13	If P(A) denotes the probability of an event A, then (a) P(A) < 0 (b) P(A) > 1 (c) 0 ≤ P(A) ≤ 1 (d) –1 ≤ P(A) ≤ 1	1
14	In Δ ABC, right-angled at B, AC = 25 cm, BC = 7 cm. The value of tan C is: (a) 12/7 (b) 24/7 (c) 20/7 (d) 7/24	1
15	HCF of (2^3 x 3^2 x 5), (2^2 x 3^3 x 5^2) and (2^4 x 3 x 5^3 x 7) is (a) 60 (b) 48 (c) 30 (d) 105	1
16	If the equation $9x^2 + 6kx + 4 = 0$ has equal roots then k = ? (a) -2 or 0 (b) 0 only (c) 2 or 0 (d) 2 or -2	1
17	The angle of depression of a car, standing on the ground, from the top of a 75 m tower, is 30°. The distance of the car from the base of the tower (in metres) is (a) 25√3 (b) 75√3 (c) 150 (d) 50√3	1
18	If one equation of a pair of dependent linear equations is -3x+5y-2=0. The second equation will be: (a) -6x + 10y - 4 = 0 (b) 6x - 10y - 4 = 0 (c) 6x + 10y - 4 = 0 (d) -6x + 10y + 4 = 0	1
19	**Direction: In the question number 19 & 20, A statement of Assertion (A) is followed by a statement of Reason(R) . Choose the correct option** **Assertion (A):** L.C.M. and H.C.F. of a and 20 are 100 and 10 respectively, then a = 50. **Reason (R):** L.C.M x H.C.F. = First number x Second number (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A). (b) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A). (c) Assertion (A) is true but Reason (R) is false. (d) Assertion (A) is false but Reason (R) is true.	1
20	**Assertion (A):** If two triangles are similar and have an equal area, then they are congruent. **Reason (R):** Corresponding sides of two triangles are equal, then triangles are congruent. (a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A) (b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of Assertion (A) (c) Assertion (A) is true but reason(R) is false. (d) Assertion (A) is false but reason(R) is true.	1
	SECTION-B **Questions 21 to 25 carry 2M each**	
21	Solve for x and y: 71x + 37y = 253, 37x + 71y = 287	2
22	α, β are the zeroes of the quadratic polynomial p(x) = $x^2 - (k+6)x - 2(2k-1)$. Find	2

	the value of k, if $\alpha + \beta = \frac{1}{3}\alpha\beta$.	
23	A group consists of 12 persons, out of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest.	2
24	Two concentric circles are of radii 6.5 cm and 2.5 cm. Find the length of the chord of the larger circle which touches the smaller circle. **OR** From an external point P, tangents PA and PB are drawn to a circle with center O. CD is the tangent to the circle at a point E touches PA and PB at C, D respectively. If PA = 14cm, find the perimeter of ΔPCD.	2
25	The mid-point of line segment joining A (2a, 4) and B (-2, 3b) is (1, 2a +1). Find the value of a and b? **OR** In what ratio does the point P(2,5) divide the join of A (8,2) and B(-6, 9)?	2
	SECTION-C **Questions 26 to 31 carry 3 marks each**	
26	Prove that (cosec A – sin A)(sec A – cos A)(tan A + cot A) = 1	3
27	Anuj had some chocolates, and he divided them into two lots A and B. He sold the first lot at the rate of ₹2 for 3 chocolates and the second lot at the rate of ₹1 per chocolate, and got a total of ₹400. If he had sold the first lot at the rate of ₹1 per chocolate, and the second lot at the rate of ₹4 for 5 chocolates, his total collection would have been ₹460. Find the total number of chocolates he had.	3
28	Prove that 7-2√5 is and irrational number **OR** If two positive integers p and q are written as $p = a^2b^3$ and $q = a^3b$, a and b are a prime number then. Verify LCM (p, q) x HCF (p, q) = p x q	3
29	The angle of elevation of an aeroplane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the plane is flying at a constant height of 1500√3m, find the speed in km/hr of the plane.	3
30	Prove that a parallelogram circumscribing a circle is a rhombus **OR** If a circle touches the side BC of a triangle ABC at P and extended sides AB and AC at Q and R, respectively, prove that $AQ = \frac{1}{2}(AB + BC + AC)$  	3
31	A heap of rice is in the form of a cone of diameter 9 m and height 3.5 m. Find the volume of the rice. How much canvas cloth is required to just cover the heap?	3

	SECTION-D **Questions 32 to 35 carry 5M each**	
32	To fill a swimming pool two pipes are used. If the pipe of larger diameter used for 4 hours and the pipe of smaller diameter for 9 hours, only half of the pool can be filled. Find, how long it would take for each pipe to fill the pool separately, if the pipe of smaller diameter takes 10 hours more than the pipe of larger diameter to fill the pool? **OR** A plane left 30 minutes late than its scheduled time and in order to reach the destination 1500 km away in time, it had to increase its speed by 100 km/h from the usual speed. Find its usual speed	5
33	Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. Using the above theorem prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non-parallel sides in the same ratio.	5
34	In a circle of radius 21cm, an arc subtends an angle of 60° at the centre. Find (i) the length of the minor arc (ii) the area of the sector (iii) the area of the minor segment (iv) the area of the major segment (v) the length of the major arc. **OR** The below figure depicts a racing track whose left and right ends are semicircular.  The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find : (i) the distance around the track along its inner edge (ii) the area of the track.	5
35	Find the values of x and y if the median for the following data is 31.	5

Class interval	0-10	10-20	20-30	30-40	40-50	50-60	Total
Frequency	5	x	6	y	6	5	40

	SECTION-E (Case Study Based Questions) **Questions 36 to 38 carry 4M each**	

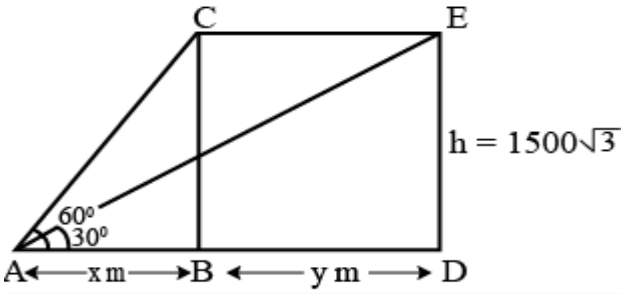
36	The school auditorium was to be constructed to accommodate at least 1500 people. The chairs are to be placed in concentric circular arrangement in such a way that each succeeding circular row has 10 seats more than the previous one.  (i) If the first circular row has 30 seats, how many seats will be there in the 10th row? (ii) For 1500 seats in the auditorium, how many rows need to be there? **OR** If 1500 seats are to be arranged in the auditorium, how many seats are still left to be put after 10th row? (iii) If there were 17 rows in the auditorium, how many seats will be there in the middle row?	4
37	The below picture are few natural examples of parabolic shape which is represented by a quadratic polynomial. A parabolic arch is an arch in the shape of a parabola. In structures, their curve represents an efficient method of load, and so can be found in bridges and in architecture in a variety of forms.     1. In the standard form of quadratic polynomial, $ax^2 + bx + c$, a, b and c are a) All are Polynomials. b) All are rational numbers. c) 'a' is a non zero real number and b and c are any real numbers. d) All are integers. 2. If the roots of the quadratic polynomial are equal, where the discriminant$D = b^2 + 4ac$, then a) D > 0 b) D < 0 c) D ≥ 0 d) D = 0	4

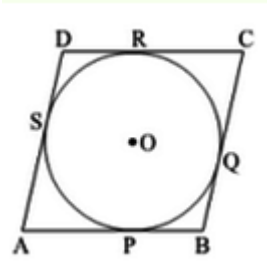
	3. If α and 1/α are the zeroes of the quadratic polynomial $2x^2 - x + k$, then k is a) 4 b) 1/4 c) –1/4 d) 2 4. The graph of $x^2 + 1 = 0$ a) Intersects x-axis at two distinct points. b) Touches x-axis at a point. c) Neither touches nor intersects x-axis. d) Either touches or intersects x- axis.	
38	Abhinav Bindra is retired sport shooter and currently India's only individual Olympic gold medalist. His gold in the 10-meter air rifle event at the 2008 Summer Olympics was also India's first Olympic gold medal since 1980. He is the first Indian to have held concurrently the world and Olympic titles for the men's 10-meter air rifle event, having earned those honors at the 2008 Summer Olympics and the 2006 ISSF World Shooting Championships. Bindra has also won nine medals at the Commonwealth Games and three gold medals at the Asian Games. A circular dartboard has a total radius of 8 inch, with circular bands that are 2 inch wide, as shown in figure. Abhinav is still skilled enough to hit this board 100% of the time so he always score at least two points each time he throw a dart. Assume the probabilities are related to area, on the next dart that he throw. (i) What is the probability that he score at least 4? (ii) What is the probability that he score at least 6? (iii) What is the probability that he hit bull's eye? (iv) What is the probability that he score exactly 4 points? 	4

MARKING SCHEME
MATHEMATICS STANDARD(041)
SAMPLE PAPER-2

Q. NO	ANSWER	MARKS
1	(c) $2\sqrt{3}$	1
2	(d) -6	1
3	(c) $\sqrt{34}$	1
4	(d) 0.77	1
5	(a) 1 : 2	1
6	(b) -5/4	1
7	(a) 3	1
8	(d) 2520	1
9	(c) Mean = $\frac{1}{2}$ (Mode – 3 x Median)	1
10	(c) 9	1
11	(a) 3/26	1
12	(b) 3	1
13	(c) $0 \leq P(A) \leq 1$	1
14	(b) 24/7	1
15	(a) 60	1
16	(d) 2 or -2	1
17	(b) 75√3	1
18	(c) 6x + 10y - 4 = 0	1
19	(a)	1
20	(d)	1
21	71x + 37y = 253…….(i) 37x + 71y = 287…….(ii) By adding and subtracting we get x + y = 5 x - y = -1 by solving we get x = 2, y = 3	 1 1
22	a = 1, b = $-(k+6)$,c = $-2(2k-1)$ α + β = $\frac{1}{3}$ αβ => $k+6 = \frac{-2(2k-1)}{3}$ => k = $\frac{-16}{7}$	1 1
23	A group consists of 12 persons, out of which 3 are extremely patient, other 6 are extremely honest and rest are extremely kind. A person from the group is selected at random. Assuming that each person is equally likely to be selected, find the probability of selecting a person who is (i) extremely patient (ii) extremely kind or honest. (i) Probability of selecting a person who is extremely patient = $\frac{1}{4}$ (ii) Probability of selecting a person who is extremely kind or honest = $\frac{6+3}{12} = \frac{3}{4}$	1 1
24	For correct fig. For correct length of cord as 12 cm **OR**	½ 1½ ½

	For correct fig. For finding correct perimeter of ΔPCD as 24 cm	1½
25	The mid-point of line segment joining A (2a, 4) and B (-2, 3b) is $\left(\frac{2a-2}{2}, \frac{4+3b}{2}\right)$ A/C, $\frac{2a-2}{2} = 1$, $\frac{4+3b}{2} = 2a+1$ For solving a = 2, b = 2 **OR** For, $\frac{-6k+8}{k+1} = 2$ For finding k = 3/4	½ ½ 1 1 1
26	LHS $(\csc A - \sin A)(\sec A - \cos A)(\tan A + \cot A)$ $= (\frac{1}{\sin A} - \sin A)(\frac{1}{\cos A} - \cos A)\,(\frac{sinA}{cosA} + \frac{cosA}{sinA})$ $= using\ secA = \frac{1}{cosA}\ and\ cosecA = \frac{1}{sinA}\ and \frac{sinA}{cosA} = tanA$ $= \left(\frac{1-\sin^2 A}{sinA}\right) \times \frac{1-\cos^2 A}{cosA} \times \frac{Sin^2A + \cos^2 A}{sinAcosA}$ $= \frac{\cos^2 A}{sinA} \times \frac{\sin^2 A}{cosA} \times \frac{1}{sinAcosA}$ = 1 = RHS Hence proved.	1 1 1
27	Let the chocolates in the first lot = x and the chocolates in the second lot =y The total number of chocolates = x + y It is given that the price of 3 chocolates is 2 Rs So, The price of x chocolates = $\frac{2}{3}x$ The price of y chocolates = 1 × y = y Rs The sum of the cost of both chocolates is 400 Rs So, $\frac{2}{3}x + y = 400$ $\Rightarrow 2x + 3y = 1200$ ……(i) Similarly, In second case Cost of x chocolates = $1 \times x = x\ Rs$ Cost of y chocolates = $\frac{5}{4}y$ Now, the sum of the cost of both types of chocolates is 460 Rs. So, $x + \frac{4}{5}y = 460$ $5x + 4y = 2300$ ……(ii) Solve equation (i) and (ii) $x = 300\ \&\ y = 200$ Hence total no of chocolates = 300+200= 500	1 1 1 1

28	Let us assume 7-2√5 is rational. Let 7-2√5 = a/b, where a, b are integers and b ≠ 0 -2√5 = (a/b) - 7 => -2√5 = (a - 7b)/b => √5 = (a - 7b)/(-2b) => √5 = (7b - a)/2b Since , a,b are integers , (7b-a)/2a is rational , and so √5 is rational. This contradicts the fact that √5 is irrational . Hence, 7 - 2√5 is irrational. OR H.C.F.(p,q)= a^2b L.C.M.(p,q)= a^3b^2 L.C.M.(p,q)×H.C.F.(p,q) = a^5b^3 And, pq = p^5b^3 Therefore, L.C.M.(p,q)×H.C.F.(p,q)=pq.	 1 1 1 ½ ½ 1 1
29	For correct figure Let BC be the height at which the aeroplane flying. Then, BC = 1500√3m Let AB = x m, BD = y m So, AD = (x + y) m In △ABC, tan 60∘=BC/AB √3=1500√3/x [∵ tan 60∘=√3] x=1500 m ...(i) In △EAD, $\frac{1}{\sqrt{3}} = \frac{1500\sqrt{3}}{x+y}$ [∵ tan 30∘=1/√3] x+y =1500×3		½ ½ ½ ½

	y=4500−1500=3000m Speed of airplane = Distance/Time $= \frac{3000}{15}$ = 200 m/s $= 200 \times \frac{18}{5}$ km/hr = 720 km/h Speed of airplane is 720 km/hr.	1
30	Given ABCD be a parallelogram circumscribing a circle with centre O. To Prove : ABCD is a rhombus. We know that the tangents drawn to a circle from an exterior point are equal is length. ∴ AP = AS, BP = BQ, CR = CQ and DR = DS. AP+BP+CR+DR = AS+BQ+CQ+DS (AP+BP) + (CR+DR) = (AS+DS) + (BQ+CQ) ∴ AB+CD=AD+BC or 2AB=2AD (since AB=DC and AD=BC of parallelogram ABCD) ∴ AB=BC=DC=AD Therefore, ABCD is a rhombus. Or We know that the tangents drawn through an external point to a circle are equal. So, BP = BQ ---------- (1) CP = CR ---------- (2) AQ = AR ---------- (3) Perimeter of triangle ABC = AB + BC + AC From the figure, BC = BP + PC So, AB + BC + AC = AB + (BP + PC) + AC From (1) and (2), = AB + BQ + CR + AC And, AB + BQ = AQ, CR + AC = AR As the tangents drawn through an external point to a circle are equal BQ + CR = BC So, AB + BC + AC = AQ + AR From (3), AB + BC + AC = AQ + AQ AB + BC + AC = 2AQ So, AQ = 1/2(AB + BC + AC) Therefore, it is proven that AQ = 1/2(AB + BC + AC)	 1 1 1 1 1 1
31	Volume of cone= 74.25 cm^3	1

	Slant height = 5.7 cm Curved surface area= 80.62 cm^2	1 1
32	Let the time taken by larger pipe alone to fill the tank= x hrs Therefore, the time taken by the smaller pipe = x+10 hrs Let us assume that, the quantity water to be filled is 1lt. Water filled by larger pipe running for 4hrs = 4/x lt. Water filled by smaller pipe running for 9hrs= $\frac{9}{x+10}$ lt. We know that $(\frac{4}{x}) + (\frac{9}{(x+10)}) = \frac{1}{2}$ $\frac{(4x+40+9x)}{(x^2+10x)} = \frac{1}{2}$ $\frac{(13x+40)}{(x^2 10x)} = \frac{1}{2}$ $26 \times 80 = x^2 + 10x$ $x^2 - 16x - 80 = 0$ $x^2 - 20x4x - 80 = 0$ $x(x-20) + 4(x-20) = 0$ $(x4)(x-20) = 0$ $x=-4, 20$ x cannot be negative. Thus, x=20 x+10= 30 Larger pipe would alone fill the tank in 20 hrs and smaller pipe would fill the tank alone in 30 hrs Or	1 2 1 1

	Let the usual speed of plane be x km/h. Increased speed = (x + 100) km/h. ∴ Distance to cover = 1500 km. Time taken by plane with usual speed = $\frac{1500}{x}$hr. Time taken by plane with increased speed = $\frac{1500}{(100+x)}$hrs. According to the question, $\frac{1500}{x} - \frac{1500}{(100+x)} = \frac{30}{60} = \frac{1}{2}$ $1500\left[\frac{1}{x} - \frac{1}{x+100}\right] = \frac{1}{2}$ $1500\left[\frac{x+100-x}{(x)(x+100)}\right] = \frac{1}{2}$ $\frac{1500\times100}{x^2+100x} = \frac{1}{2}$ $x^2 + 100x = 300000$ $x^2 + 100x - 300000 = 0$ $x^2 + 600x - 500x - 300000 = 0$ $x(x + 600) - 500(x + 600) = 0$ $(x + 600)(x - 500) = 0$ x = -600 (Rejected) x - 500 = 0 x = 500 ∴ Usual speed of plane = 500 km/hr.	
33	For correct proof For correct proof	3 2
34	(i) Length of the Arc, APB = θ/360° × 2πr = 60°/360° × 2 × 22/7 × 21 cm = 22 cm (ii) Area of the sector, AOBP = θ/360° x πr² = 60°/360° × 22/7 × 21 × 21 cm² = 231 cm² (iii) Area of the minor segment= 41.37 cm² (iv) Area of the major segment = 1344.63 cm² (v) The length of the major arc = 110 cm OR (vi) Distance around the track along its inner edge = $\frac{2807}{7}m$ (vii) Area of the track = 4320 m2	1 1 1 1 1

35		

Classes	Frequency	c.f
$0-10$	5	5
$10-20$	x	$5+x$
$20-30$	6	$11+x$
$30-40$	y	$11+x+y$
$40-50$	6	$17+x+y$
$50-60$	5	$22+x+y$
Total	40	

For correct table — 1

1

$\Rightarrow$ 22+x+y=40 — 1

$\therefore$ x+y=18 ----- (1)

$\Rightarrow$ Here, median class $= 30-40$ — 1

$\Rightarrow l = 30,\ c.f = 11+x,\ f = y,\ h = 10,\ \frac{n}{2} = \frac{40}{2} = 20$

$\Rightarrow$ Median $= 31$

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$31 = 30 + \left[\frac{20-(11+x)}{y}\right] \times 10$$ — 1

$$1 = \frac{9-x}{y} \times 10$$

$\therefore\ 10x + y = 90$ ---- (2)

Now, subtracting equation (1) from (2), we get

$x = 8$ and

$y = 10$

36	i) 120	1

	ii) n=15 OR 750 iii) 110seats	1 2
37	1. c 2. d 3. b 4. c	1 1 1 1
38	(i) 9/16 (ii) 1/4 (iii) 3/16 (iv) 5/16	1 1 1 1

SAMPLE PAPER-3

CLASS - X

SUBJECT - MATHEMATICS (Standard)

TIME: 3 HOURS **M.M.:80**

GENERAL INSTRUCTIONS:

(a) This Question Paper has 5 Sections A, B, C, D and E.

(b) Section A has 20 MCQs carrying 01 mark each.

(c) Section B has 5 questions carrying 02 marks each.

(d) Section C has 6 questions carrying 03 marks each.

(e) Section D has 4 questions carrying 05 marks each.

(f) Section E has 3 case based integrated units of assessment (04 marks each) with sub-parts of the values of 1, 1 and 2 marks each respectively.

(g) All Questions are compulsory; However, an internal choice in 2 questions of 5 marks, 2 questions of 3 marks and 2 questions of 2 marks has been provided. An internal choice has been provided in the 2 marks question of section E.

(h) Draw neat figures wherever required take $\pi = 22/7$ wherever required if not stated.

(i) Use of Calculator is Prohibited.

Section- A

1. The ratio in which the line segment joining the points **(-3,5)** and **(4,-9)**, is divided by **(2,-5)** is

   a) 2 : 3 b) 1:3 c) 5 : 2 d) 3 : 1

2. The value of x and y; $\mathbf{x + 2y = 9}$, $\mathbf{2x - y = 8}$

   a) 0, 0 b) 5,2 c) 5, 3 d) 0, 1

3. If $\mathbf{S_n = 5n^2 + 3n}$, then its common difference is :-

   a) 18 b) 8 c) 10 d) 26

4. Determine the upper limit of the **modal class** of the following frequency distribution :-

Class	0-5	6-11	12-17	18-23	24-29
Frequency	13	10	15	8	11

   a) 17 b) 19.5 c) 18 d) 17.5

5. The nature of the roots of the quadratic equation $\mathbf{2x^2 + x + 4 = 0}$ is :-

   a) no real roots b) real roots c) equal roots d) none of these

6. If the distance between the points **(4,p)** and **(1,0)** is **5** units, then the value of **p** is :-

a) 3 b) ± 4 c) 5 d) -3

7. A quadratic polynomial, whose zeroes are -3 and 4 is :-

a) $x^2 + x + 5$ b) x^2-x+6 c) x^2-x-12 d) x^2+2x-6

8. If $\tan\theta + \cot\theta = 2$, then the value of $\tan^2\theta + \cot^2\theta$ is :

a) 2 b) 3 c) 4 d) 5

9. If the lines represented by $3x+2py = 2$ and $2x + 5y + 1 = 0$ are parallel, then the value of p is:-

a) $\frac{15}{4}$ b) 13 c) 12 d) $\frac{19}{2}$

10. The distance between two parallel tangents to a circle of radius 5 cm is :-

a) 5 cm b) 8 cm c) 10 cm d) 9 cm

11. If the area of a sector of a circle is $\frac{5}{18}$ to the area of the circle, then find the angle subtended by minor arc at the centre.

a) 10^0 b) 100^0 c) 25^0 d) 60^0

12. If x = 3 is a root of quadratic equation $kx^2 - kx - 3 = 0$, then the value of k is :

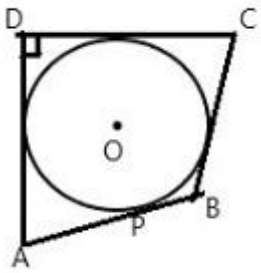
a) $\frac{3}{2}$ b) $\frac{1}{2}$ c) 2 d) $\frac{5}{2}$

13. Write the exponent of 3 in the prime factorization of 1944.

a) 3 b) 41 c) 5 d) 4

14. In the figure, $\angle ADC = 90^0$, BC = 38 cm, CD = 28 cm and BP = 25 cm, then radius of the circle is :-

a) 20 cm b) 15 cm c) 16 cm d) 18 cm



15. Two different dice are rolled together, the probability of getting a sum of 10 of the numbers on the two dice is

a) $\frac{2}{13}$ b) $\frac{5}{14}$ c) $\frac{1}{12}$ d) $\frac{1}{13}$

16. The value of 'a', if HCF (a,18) = 2 and LCM (a,18) = 36, is :-

a) 2 b) 5 c) 7 d) 4

17. If 6 times the 6th term of an A.P. is equal to 9 times the 9th term, then its 15th term will be :-

a) 10 b) 31 c) 22 d) 0

18. If α and β are the zeros of the polynomial: $px^2 - 2x + 3p$ and $\alpha + \beta = \beta$. then the value of p is

a) $\frac{-2}{3}$ b) $\frac{2}{3}$ c) $\frac{1}{3}$ d) $\frac{-1}{3}$

Assertion – Reason Type Questions :

Directions for questions 19 & 20 : In question number 19 and 20 , a statement of **Assertion (A)** is followed by a statement of **Reason (R)** . Choose the correct option –

a) Both Assertion (A) and Reason(R) are true and Reason(R) is the correct explanation of Assertion(A).

b) Both Assertion (A) and Reason(R) are true and Reason(R) is not the correct explanation of Assertion(A).

c) Assertion (A) is true but Reason(R) is false.

d) Assertion (A) is false but Reason(R) is true.

19. **Assertion (A) :** If two angles of any triangle are equal to the corresponding two angles of another triangle, then the third angle is not necessarily equal.

Reason (R) : The sum of three angles of any triangle is equal to 180^0

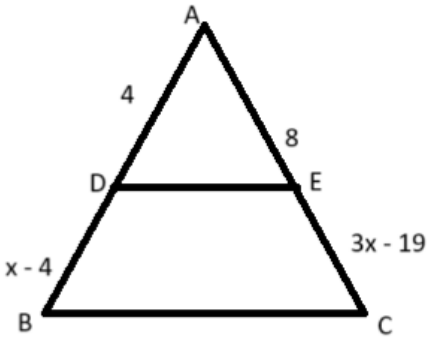
20. **Assertion (A)** : A cylinder and a right circular cone are having the same base and same height, then the volume of cylinder is three times the volume of cone.

Reason (R) : If the radius of cylinder is doubled and height is halved then the volume will be doubled.

Section B

21. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

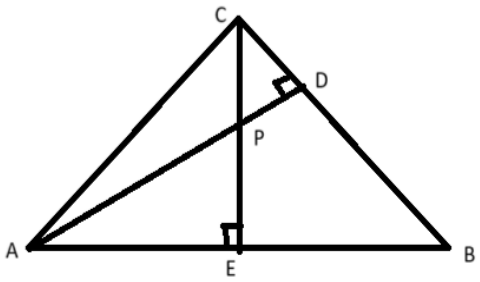
22. In the adjoining figure, find x if DE∥BC



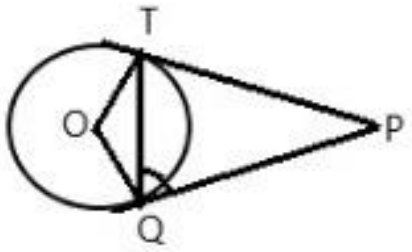
OR

In the adjoining figures altitudes AD and CE of ΔABC interest each other at P. show that $\frac{AE}{CD} = \frac{EP}{DP}$

.



23. In the adjoining figure, PT and PQ are two tangents from external point P. If $\angle PQT = 70^0$, find $\angle TOQ$.



OR

Prove that the lengths of tangents drawn from an external point to a circle are equal.

24.

Daily Income	No. of Workers
Less than 120	12
Less than 140	26
Less than 160	34
Less than 180	40
Less than 200	50

Based on the above information, find the lower limit of the median class.

25. A horse is placed for grazing inside a rectangular field of 40 m by 36 m and is tied to one corner by a 14 m long rope. How much area it can graze?

Section C

26. If $\sin\theta = \frac{12}{13}$, find the value of $\frac{sin^2\theta - Cos^2\theta}{2\sin\theta\cos\theta} - \frac{1}{tan^2\theta}$

OR

$\frac{\tan A}{1+Sec\ A} - \frac{\tan A}{1-Sec\ A} = 2\ \text{cosec A}$

27. Prove that $6-2\sqrt{5}$ is irrational.

OR

On a morning walk, three persons step off together and their step measures 40cm, 42cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps ?

28. Solve for x.

$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$; $x \neq 0, 1, 2$

29. If α *and* β are the zeroes of the polynomial $x^2-8x+15$, then find a quadratic polynomial whose zeroes are 3α and 3β.

30. Determine the value of k so that the following pairs of equations are inconsistent.

$(3K + 1)x + 3y - 2 = 0$

$(K^2+1)x + (K-2)y - 5 = 0$

31. If two tangents inclined at an angle of 60^0 are drawn to a circle of radius 3cm, then find the length of each tangent.

Section - D

32. Find the mean, median and mode for the data.

Class	0-10	10-20	20-30	30-40	40-50	50-60	60-70
Frequency	5	8	15	20	14	8	5

OR

The median of the data is 28. Find the values of x and y, if the total frequency is 50

Marks	0-10	10-20	20-30	30-40	40-50
No of students	5	x	15	y	6

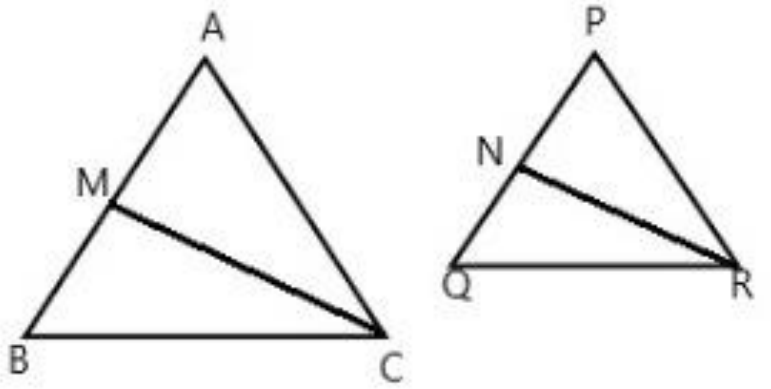
33. A tent is in the shape of a right circular cylinder surmounted by a cone. The total height and the diameter of the base are 13.5m and 28m, respectively. If the height of the cylindrical portion is 3m, find the total surface area of the tent.

OR

A toy is in the shape of a right circular cone surmounted by a hemisphere. The radius of the hemisphere and cone is 5 cm. Find the surface area of the toy if the total height of the toy is 17 cm.

34. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that $\tan\theta = 1$ or $\frac{1}{2}$.

35. In figure, CM and RN are respectively the medians of ΔABC and ΔPQR. If ΔABC ~ΔPQR, prove that :



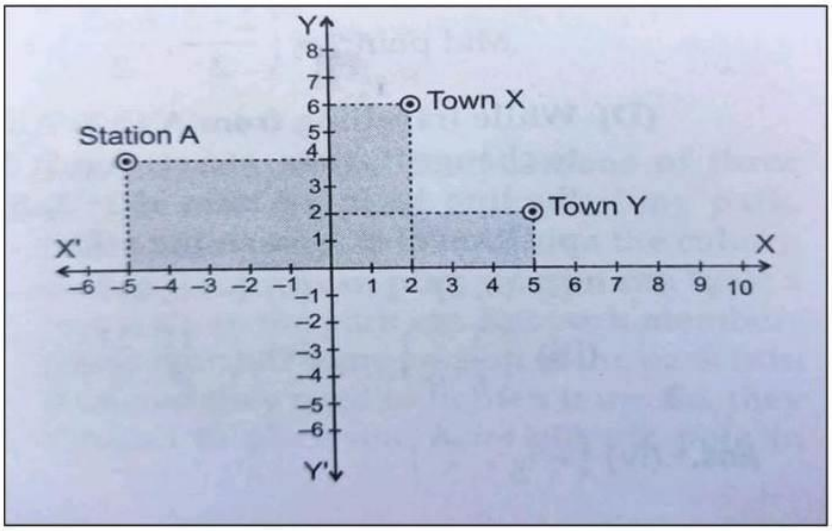
i) $\Delta\, AMC \sim \Delta$ PNR

ii) $\frac{CM}{RN} = \frac{AB}{PQ}$

iii) Δ CMB $\sim \Delta\, RNQ$

Section -E

36. Two friends Dalvin and Alice work in the same office in Toronto. In the Christmas vacation, they both decided to go to their home towns represented by Town X and Town Y. Town X and Town Y are connected by trains from the same station A in Toronto. The situation of Town X , Town Y and station A is shown on the coordinate axis.



Based on the given situation, answer the following questions :

i) What is the distance that Dalvin has to travel to reach his hometown X ?

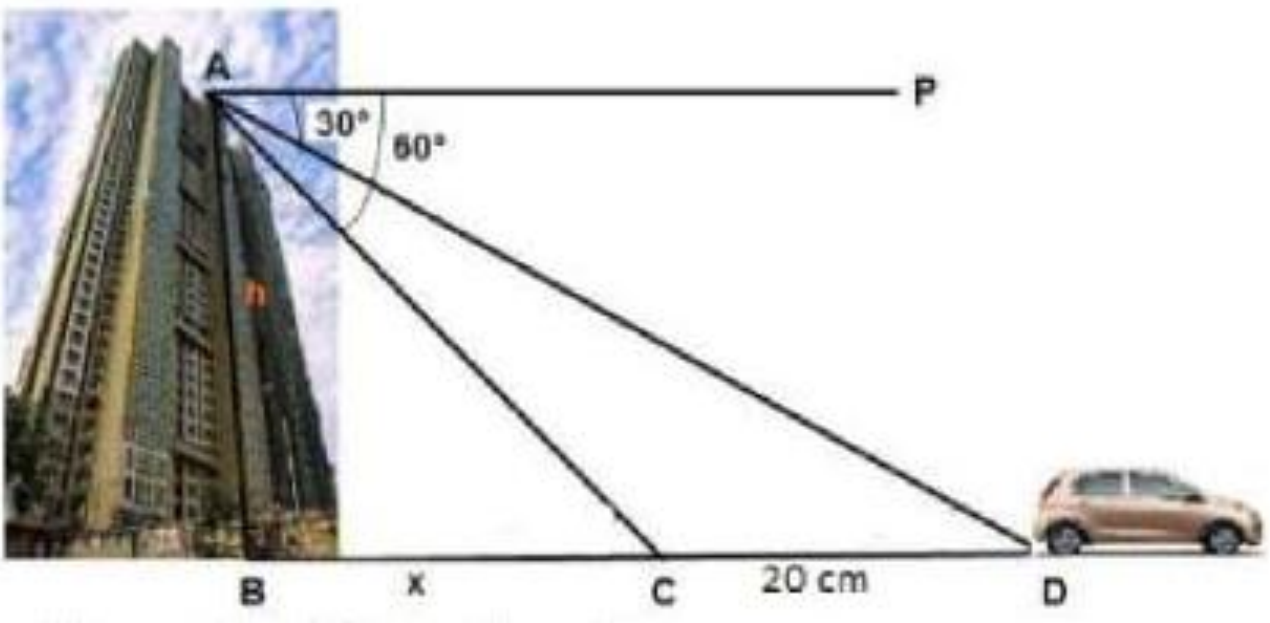
ii) What is the distance that Alice has to travel to reach her hometown Y ?

iii) While travelling from A to Y, Alice had to change the train, at a station, it divides the line AY in the ratio of 2:3, find the coordinates of position of that station.

OR

iii) In what ratio, Y - axis divides the line segment joining A and Y? Also find the coordinates of point of intersection.

37. Vijay lives in the flat in a multi-story building. Initially, his driving was rough so his father keeps eye on his driving. Once he drives from his house to Faridabad. His father was standing on the top of the building at point A as shown in the figure. At point C, the angle of depression of a car from the building was 60^0, After accelerating 20 m from point C, Vijay stops at point D to buy ice cream and the angle of depression changed to 30^0.



i) Find the value of x.

ii) Find the height of the building AB.

OR

ii) Find the distance between top of the building and a car at position C ?

iii) Find the distance between top of the building and a car at position D ?

38. Saving money is a good habit and it should be inculcated in children from the beginning. Rajiv gets pocket money from his father everyday. Out of pocket money, he saves Rs. 2.75 on 1st day and on each succeeding day he increases his saving by 25 paise.

i) On which day he saves Rs. 7.75?

ii) Find the difference between the amount saved by Rajiv on 25th day and 14th day.

iii) In how many days, he will save Rs. 38.75?

OR

iii) Find the sum of amount saved on first 20 days.

MARKING SCHEME_SAMPLE PAPER-3

CLASS - X

SUBJECT - MATHEMATICS (Standard)

S. No.	ANSWER	Marks
	SECTION- A	
1.	c	1
2.	b	1
3.	c	1
4.	a	1
5.	a	1
6.	b	1
7.	c	1
8.	a	1
9.	a	1
10.	c	1
11.	b	1
12.	b	1
13.	c	1
14.	b	1
15.	c	1
16.	d	1
17.	d	1
18.	b	1
19.	d	1
20.	b	1
	SECTION-B	
21.	$formula\ for\ probability\ ,\quad \frac{7}{8}$	1+1
22.	BPT Theorem , ratio , $x = 11$ OR Identification of triangles , Similarity of triangles ΔAPE and ΔCPD , C.P.S.T	½ + 1 + ½

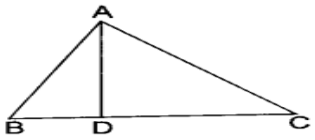
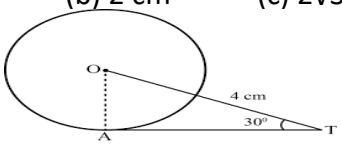
23.	Theorem , ASP , $\angle TOQ = 140°$	1+1
24.	Modification class interval , lower limit of median class = 120	1 + 1
25.	Formula for area of quadrant $= \frac{1}{4}\pi r^2 = 154\text{m}^2$	1+1
26.	SECTION-C Use of T-Ratios , $\frac{589}{720}$ OR Use of Trigonometric Identities to proof	3
27.	Contradiction method OR L.C.M(40,42,45) = 2520cm	3
28.	LCM and solve X = 3 , $\frac{4}{3}$	3
29.	Roots 3 and 5 x^2-24x + 135 = 0	1+ ½ +1½
30.	Use $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, K = -1	1+1+1
31.	Use T-Ratios , Length of each tangent = $3\sqrt{3}$ cm h	1+2
32.	SECTION-D Mean = 35.26 Median 34.75 Mode = 34.5 OR x =13 y = 11	5
33.	Use formula , 1034 m^2 OR 171.4 cm^2	5
34.	Proof Use Trigonometric identities $\sin^2\theta + \cos^2\theta = 1$ and solve using	5

	algebraic identities	
35.	Proof	5
36.	SECTION-E (i) $\sqrt{53}\ units$ (ii) $\sqrt{104}\ units$ (iii) $(-1, \frac{16}{5})$ OR 1:1	1+1+2
37.	(i) 10m (ii) $10\sqrt{3}$ m OR 20m (iii) $20\sqrt{3}$	1+1+2
38.	(i) 21st day (ii) Rs 2.75 (iii) 10days OR Rs 102.5	1+1+2

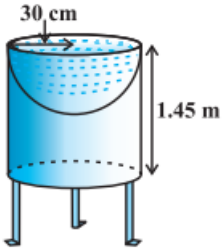
Sample Question Paper -4

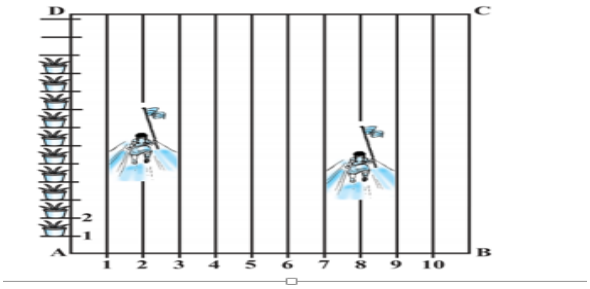
Class X
Basic Mathematics (241)

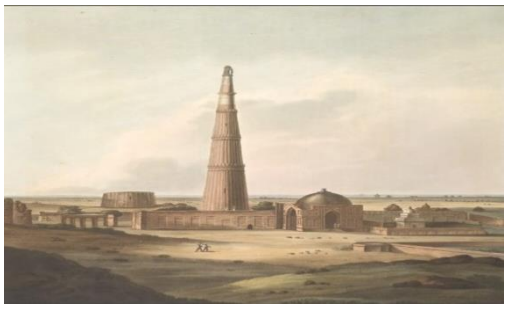
Time Allowed: 3 Hrs **Maximum Marks: 80**

Q. no	Section A	Marks
1.	Which of the following cannot be the probability of an event? (A) 2 /3 (B) –1.5 (C) 15% (D) 0.7	1
2.	Volume of two spheres are in the ratio of 64 : 27 , then ratio of their radii are a) 4/3 b) 3/9 c) 4/9 d) 1/4	1
3.	In ∠BAC = 90° and AD ⊥ BC. Then  (a) BD.CD = BC² (б) AB.AC = BC² (c) BD.CD = AD² (d) AB.AC = AD²	1
4.	The value of 'k' for which the system of equation x + 2y- 3=0 and 5x + ky + 7=0 has no solutions is (A) k=10 (B) k=6 (C) k=3 (D) k=1	1
5.	If tan θ = 1, then the value of sec θ + cosec θ is: (a) 3√2 (b) 4√2 (c) 2√2 (d) √2	1
6.	If ΔABC~ΔDEF such that 2AB = DE and BC = 8cm , then EF is equal to a) 12cm b) 16cm c) 8 cm d) 18cm	1
7.	which of the following equations has the sum of its roots as 3? (A) $2x^2 - 3x + 6 = 0$ (B) $-x^2 + 3x - 3 = 0$ (C) $\sqrt{2}x^2 - \frac{3}{\sqrt{2}}x + 1 = 0$ (D) $3x^2 - 3x + 3 = 0$	1
8.	A quadratic polynomial, whose zeroes are –3 and 4, is (A) $x^2 - x + 12$ (B) $x^2 + x + 12$ (C) $\frac{x^2}{2} - \frac{x}{2} - 6$ (D) $2x^2 + 2x - 24$	1
9.	LCM of $(2^3 \times 3 \times 5)$ and $(2^4 \times 5 \times 7)$ is (A) 40 (B) 560 (C) 1120 (D) 1680	1
10.	In the given figure, AT is a tangent to the circle with centre O such that OT = 4 cm and ∠OTA = 30° , then AT = (a) 4 cm (b) 2 cm (c) 2√3 cm (d) 4√3 cm  	1
11.	If ΔABC is right angled at C , then the value of cos (A +B) is: A) 0 B) 1 C)1/2 D) not defined	1
12.	In a circle of radius 14 cm, an arc subtends an angle of 90^0 at the center , then the areaof the sector is: A) 71 cm^2 B) 76 cm^2 C) 77 cm^2 D) 154 cm^2	1
13.	If the HCF of 65 and 117 is expressible in the form 65*m* – 117, then the value of *m* is (A) 4 (B) 2 (C) 1 (D) 3	1
14.	If the distance between the points (2, –2) and (–1, *x*) is 5, one of the values of *x* is (A) –2 (B) 2 (C) –1 (D) 1	1

15.	The lower limit of modal class is of the following data is: Class interval | 0-10 | 10-20 | 20-30 | 30-40 | 40-50 Frequency | 5 | 8 | 13 | 7 | 6 a) 10 b) 20 c) 50 d) 30	1
16.	The median and mode respectively of a frequency distribution are 26 and 29 , Then the mean is : a) 27.5 b) 24.5 c) 28.5 d) 25.8	
17.	The top of two towers of height x and y standing on a level ground subtends an angle of 30^0 and 60^0 respectively at the center of the line joining their feet, then x : y = a) 1 : 2 b) 1 : 3 c) 1 : 4 d) 1 : 1	1
18.	If the equation $x^2 + 2(k+2)x + 9k = 0$ has equal roots then k =? (A) 1 or 4 (B) – 1 or 4 (C) 1 or – 4 (D) – 1 or – 4	1
19.	Assertion (A): The number 5^n cannot end with digit 0, where n is a natural number. Reason(R) :A number ending with 0 should have 2 and 5 as its prime factorization a)Both assertion and reason are true and reason is the correct explanation of assertion. b) Both assertion and reason are true but reason is not the correct explanation of assertion. c) Assertion is true but reason is false. d) Assertion is false and reason is true.	1
20.	Assertion (A): the value of y is 6, for which the distance between the points p(2, -3) and q(10, y) is 10. Reason (R): Distance between the two points $P(x_1,y_1)$ and $Q(x_2,y_2)$ is given by $PQ = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$. a)Both assertion and reason are true and reason is the correct explanation of assertion b) Both assertion and reason are true but reason is not the correct explanation of assertion c) Assertion is true but reason is false. d) Assertion is false and reason is true.	1
	Section B	
21.	Find the diameter of a circle whose area is equal to the sum of the areas of two circles of radii 40cm and 9cm.	2
22.	Solve 2x + 3y = 11 and 2x – 4y = – 24 and hence find the value of 'm' for which y = mx + 3.	2
23.	In figure, if AD = 6cm, DB = 9cm, AE = 8cm and EC = 12cm and ∠ADE = 48˚. Find ∠ABC.  OR S and T are points on sides PR and QR of Δ PQR such that ∠ P = ∠ RTS. Show that Δ RPQ ~ Δ RTS.	2
24.	Find the area of the sector of a circle with radius 4 cm and of angle 30°. Also, find the	2

	area of the corresponding major sector. OR Find the area of a quadrant of a circle whose circumference is 22 cm.	
25.	If tan (A + B) = $\sqrt{3}$ and tan (A – B) = $1/\sqrt{3}$; 0° < A + B ≤ 90°; A > B, find A and B	2
	SECTION C	
26.	A lending library has a fixed charge for the first three days and an additional charge for each day thereafter. Saritha paid Rs.27 for a book kept for seven days, while Susy paid Rs.21 for the book she kept for five days. Find the fixed charge and the charge for each extra day. OR If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes 1/ 2 if we only add 1 to the denominator. What is the fraction?	3
27.	Prove that $\sqrt{3}$ is irrational.	3
28.	Find the zeroes of the quadratic polynomial p(x) = x^2 + 7x + 12 and verify the relationship between the zeroes and its coefficient.	3
29.	Prove that $(\sin A + \operatorname{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$	3
30.	Prove that the parallelogram circumscribing a circle is a rhombus. OR The lengths of tangents drawn from an external point to a circle are equal.	3
31.	One card is drawn from a pack of 52 cards. What is the probability of getting? (ii) An ace? (iii) A red card? (iv) A face card?	3
	SECTION D	
32.	ABCD is a trapezium in which AB || DC and its diagonals intersect each other at the point O. Show that AO/ CO= BO/ DO OR If AD and PM are medians of triangles ABC and PQR, respectively where Δ ABC ~ Δ PQR, prove that AB /PQ = AD/ PM	5
33.	Mayank made a bird-bath for his garden in the shape of a cylinder with a hemispherical depression at one end (see Fig) The height of the cylinder is 1.45 m and its radius is 30 cm. Find the total surface area of the bird-bath. (Take π = 22 7)  OR A solid iron pole consists of a cylinder of height 220 cm and base diameter 24 cm, which is surmounted by another cylinder of height 60 cm and radius 8 cm. Find the mass of the pole, given that 1 cm3 of iron has approximately 8g mass. (Use π = 3.14)	5

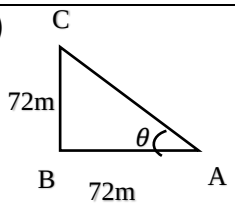
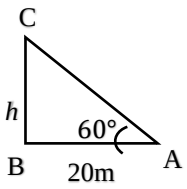
<table>
<tr><td>34
.</td><td>If the median of the distribution given below is 28.5, find the values of x and y,
<table>
<tr><td>Class interval</td><td>0-10</td><td>10-20</td><td>20-30</td><td>30-40</td><td>40-50</td><td>50-60</td><td>Total</td></tr>
<tr><td>Frequency</td><td>5</td><td>x</td><td>20</td><td>15</td><td>y</td><td>5</td><td>60</td></tr>
</table></td><td>5</td></tr>
<tr><td>35.</td><td>A train travels 360 km at a uniform speed. If the speed had been 5km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.
Or
Solve for x ; $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ (a, b, x $\neq$ 0)</td><td>5</td></tr>
<tr><td>36.</td><td>CASE STUDY: Answer the following questions.
Your friend Veer wants to participate in a 200 m race. He can currently run that distance in 51 seconds and with each day of practice it take him 2 seconds less. He wants to do in 31 seconds
 
(i) Write an AP based on above information, to represent the situation.
(ii) What is the minimum number of days he needs to practice till his goal is achieved?</td><td>2+2</td></tr>
<tr><td>37.</td><td>A City school is organizing Annual sports event in a rectangular shaped ground ABCD the tracks are being marked with a gap of I’m each in the form of straight lines. 120 flower pots are placed with a distance of 1 m each along AD. Shruti runs 1/ 3 rd of the distance in the second line along AD and post her flag. Saanvi runs 1/ 5 th the distance AD in the eighth line and posts her flag
 

(I) What is coordinates of the point where Shruti and sannavi posts their flags.
(II) Find the distance between the two flags</td><td>2+2</td></tr>
<tr><td>38.</td><td>A group of visitors visited Qutab Minar on an educational trip. They had interest in history behind Qutab Minar. The guide narrated that Qutab Minar’s , official name is Vishnu stambha, minaret and "victory tower" that forms part of the Qutab complex, It is one of the most visited tourist spots in the city,
 
mostly built between 1199 and 1220. The teacher also said that the Qutab miner is the tallest masonry tower in India, measuring 72 meters. (appx) height.(Take √3=1.73)
i) What is the angle of elevation if they are standing at a distance of 72 meters away from the monument?
ii) If the altitude of sun is at 60 degree, then the height of the vertical tower that will cast a shadow of length 20 m, is?</td><td></td></tr>
</table>

MARKING SCHEME
MATHEMATICS BASIC(241)
SAMPLE PAPER-4

Q.NO.	ANSWERS	MARKS
SECTION – A		
1.	(b) -1.5	1
2.	(a) 4/3	1
3.	(c) $BD.CD = AD^2$	1
4.	(a) $k = 10$	1
5.	(c) $2\sqrt{2}$	1
6.	(b) 16 cm	1
7.	(b)$-x^2 + 3x - 3 = 0$	1
8.	(c) $\frac{x^2}{2} - \frac{x}{2} - 6$	1
9.	(d) 1680	1
10.	(c) $2\sqrt{3}$	1
11.	(a) 0	1
12.	(d) $154cm^2$	1
13.	(b) $m = 2$	1
14.	(b) 2	1
15.	(b) 20	1
16.	(b) 24.5	1
17.	(b) 1:3	1
18.	(a) 1 or 4	1
19.	(a) Both A and R are true and R is the correct explanation of A.	1
20.	(d) A is false, R is true.	1
SECTION - B		
21.	Diameter of given circle = 41 × 2 = 82cm	2
22.	Correct value of x = - 2 Correct value of y = 5 Correct value of m = - 1	2
23.	$\frac{AD}{AB} = \frac{6}{6+9} = \frac{6}{15} = \frac{2}{5}$; $\frac{AE}{AC} = \frac{8}{20} = \frac{2}{5}$; ∠DAE=∠BAC (common angle) ⇒ ΔADE ~ ΔABC (By SAS similarity) ⇒ ∠ADE = ∠ABC = 48° (By AA similarity) **OR** Correct Proof	2
24.	Area of sector = 4.1904 cm^2 The area of the corresponding major sector = 46.09 cm2 OR r = 3.5 cm Area of quadrant = 9.625 cm^2	2
25.	A = 45° B = 15°	2
SECTION – C		
26.	Value of x = 15	3

	Value of y = 3 **OR** Fraction = $\frac{3}{5}$	
27.	Correct Proof	3
28.	x = -4, x = -3 Verification $\alpha + \beta = \frac{-b}{a} = -7$ $\alpha\beta = \frac{c}{a} = 12$ Sum of zeroes = -4 – 3 = -7 Product of zeroes = (-4)(-3) = 12	3
29.	Correct Proof	3
30.	Correct Proof Or Correct Proof	3
31.	P(ace) = $\frac{4}{52}$ P(a red card) = $\frac{26}{52}$ P(face card) = $\frac{12}{52}$	3
SECTION - D		
32.	Correct Proof Or Correct Proof	5
33.	TSA of bird – bath = 33000cm^2 = 3.3m^2 **OR** Total volume of pole = 111532.8cm^3 Mass of 1cm^3 iron = 8g Mass of 111532.8cm^3 iron = 111532.8×8 = 892262g = 892.262kg	5
34.	$x = 8\ and\ y = 7$	5
35.	Speed of the train = 40 km/h **OR** $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$ $\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$ $\frac{x-(a+b+x)}{(a+b+x)x} = \frac{b+a}{ab}$ $\frac{-(a+b)}{xa+xb+x^2} = \frac{(b+a)}{ab}$ $-ab = x^2 + bx + ax$ $x^2 + bx + ax + ab = 0$ $x(x+b) + a(x+b) = 0$	5

	$(x+b)(x+a)=0$ $x=-b, x=-a$	
SECTION – E		
36.	(i) 49, 47, … (ii) $a_n = a + (n-1)d$ $31 = 49 + (n-1)(-2)$ $\frac{-18}{-2} = n-1$ $9 = n-1 \Rightarrow n = 10$	2 2
37.	(i) Shruti(2, 40); Sannavi(8, 24) (ii) Distance between the two flags = $\sqrt{(8-2)^2+(24-40)^2}$ $= \sqrt{36+255} = \sqrt{292}$	2 2
38.	(i)  $\tan\theta = \frac{BC}{BA} = \frac{72}{72} = 1 = \tan 45°$ (ii)  In ΔABC, $\tan 60^\circ = \frac{BC}{BA}$ $\sqrt{3} = \frac{h}{20} \Rightarrow h = 20\sqrt{3}cm$	2 2

तत् त्वं पूषन् अपावृणु

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